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Networks in Conflict: Theory and Evidence from the Great War of Africa[1]

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Abstract: We study from both a theoretical and an empirical perspective how a network of military alliances and enmities affects the intensity of a conflict. The model combines elements from network theory and from the politico-economic theory of conflict. We postulate a Tullock contest success function augmented by an externality: each group's strength is increased by the fighting effort of its allies, and weakened by the fighting effort of its rivals. We obtain a closed form characterization of the Nash equilibrium of the fighting game, and of how the network structure affects individual and total fighting efforts. We then perform an empirical analysis using data on the Second Congo War, a conflict that involves many groups in a complex network of informal alliances and rivalries. We estimate the fighting externalities, and use these to infer the extent to which the conflict intensity can be reduced through (i) removing individual groups involved in the conflict; (ii) pacification policies aimed at alleviating animosity among groups.

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1 Introduction

Alliances and enmities among armed actors – be they rooted in history or in mere tactical considerations – are part and parcel of warfare.[1] In many episodes, especially in civil conflicts, they are shallow links that are not sanctioned by formal treaties or war declarations. Even allied groups retain separate agendas and pursue self-interested goals in competition with each other. The command of armed forces remains decentralized, and coordination is minimal.

Understanding the role of informal networks is important, not only for predicting outcomes, but also for implementing policies to contain or put an end to violence. These may be diplomatic initiatives promoted by international organizations to restore dialogue and reduce animosity between conflict participants, or military interventions of external forces against specific groups. Yet, with only few exceptions, the existing political and economic theories restrict attention to conflicts among a small number of players, and do not consider network aspects. In this paper we construct a theory of conflict focusing explicitly on informal networks of alliances and enmities, and apply it econometrically to the study of the Second Congo War (1998-2003) and its aftermath.

The theoretical benchmark is a *contest success function,* henceforth CSF (see, e.g., Grossman and Kim 1995, Hirshleifer 1989, Skaperdas 1992). In a standard CSF, the share of the prize accruing to each group is determined by the amount of resources (*fighting effort*) that each of them commits to the conflict. In our model, the network of alliances and enmities modifies the sharing rule of a standard CSF by introducing additional externalities. More precisely, we assume that the share of the prize accruing to group i is determined by the group's relative strength, which we label *operational performance*, henceforth OP. In turn, the OP is determined by group i's own fighting effort and by the fighting effort of its allied and enemy groups. The fighting effort of group i's allies increases group i's OP, whereas the fighting effort of its enemies decreases it. Thus, each group's fighting effort affects positively its allies' OP and negatively its enemies'. Instead, the costs of fighting are borne individually by each group. This raises a motive for strategic behavior among both enemy and allied groups. In particular, there is not even coordination between allies: all agents determine their effort in a non-cooperative way, and alliances are loose links.[2] The complex externality web affects the optimal fighting effort of all groups.

We provide an analytical solution for the Nash equilibrium of the game. Absent other sources of heterogeneity, the fighting effort of each agent hinges on a measure of network centrality which is related to the Bonacich centrality (Ballester, Calvo-Armengol and Zenou 2006). Our centrality is approximately equal to the sum of the Bonacich centrality related to the network of enmities, and the (negative-parameter) Bonacich centrality related to the network of alliances. The equilibrium share of the prize accruing to each player and the associated welfare (i.e., the share of the prize net of the fighting effort) have simple expressions. Intuitively, a group's welfare is increasing in the number of its allies and decreasing in the number of its enemies.

The ultimate goal of the theoretical analysis is to predict how the network of military alliances and rivalries affects the overall conflict intensity. This is measured by the sum of the fighting efforts

[1]Ghez (2011) distinguishes between tactical, historical and natural alliances. Tactical alliances are formed "to counter an immediate threat or adversary that has the potential to challenge a state's most vital interests" (p. 20). They are instrumental and often opportunistic in nature. Historical alliances are more resilient insofar as they hinge on a historical tradition of cooperation. However, they often remain informal. Natural alliances imply a more profound shared political culture and vision of the world (e.g., Western Europe and the U.S.). Contrary to tactical and historical alliances, natural alliances often are formalized relationships. Our study focuses on tactical and historical alliances/enmities. In our theory, natural allies can be viewed as merged actors acting in a perfectly coordinated fashion.

[2]In some historical examples alliances are more than shallow links. Our theory can incorporate strong alliances (e.g., between the U.K and the U.S. during WWII) by treating them as merged unitary groups.

of all contenders (total rent dissipation), which is our measure of the welfare loss associated with a conflict. We show that network externalities are a key driver of the escalation or containment of violence.

As an illustrative example, we analyze a "regular" network, in which the number of alliances and enmities is invariant across groups. We show that conflict intensity and rent dissipation are maximized when all groups are connected by enmity links. In this case, the outcome is a Hobbesian pre-contractual *homo homini lupus* society. When externalities are sufficiently strong, the cost of conflict may offset the social surplus in this society. To the opposite extreme, conflict and rent dissipation are minimized (and, possibly, vanish) in networks where all groups are allied, as in Rousseau's *well-ordered society* governed by the social contract. The well-ordered society may be a surprising outcome in a non-cooperative contest between self-interested agents. The crux of the result is that the marginal product of fighting effort decreases in the number of alliances, because they dilute the marginal benefit from exercising individual fighting effort. For sufficiently strong alliance externalities, the incentive to fight vanishes altogether. The standard free-riding problem has a benign effect in our model, since war effort has no social value. The peaceful outcome can be viewed as representing a society in which the system of institutional checks and balances reduces the return to opportunistic behavior.

In the second part of the paper, we perform an empirical analysis based on the structural equations of the model. We focus on the Second Congo War, sometimes referred to as the "Great African War". This is a big conflict, with an estimated death toll of 3-to-5 million lives (Autesserre 2008; Olsson and Fors 2004). It involves many groups, and a rich network of alliances and enmities.[3] To identify the network of alliances and enmities we use information from the Stockholm International Peace Research Institute (SIPRI), supplemented by information from the Armed Conflict Location Event Database (ACLED). We assume that the network is exogenous and time-invariant – an assumption that conforms with the data for the period considered (see the discussion in section 3.3) – and proxy fighting effort by the number of fighting events in which each group is involved. The estimated networks of enmities and alliances feature numerous intransitivities, showing that the Second Congo War can by no means be described as a fight between two unitary opposing camps (see Figure 6 below). Our estimation strategy exploits panel variations in the yearly fighting efforts exerted by 85 armed groups over the 1998-2010 period. Controlling for group fixed effects and time-varying unobserved heterogeneity we regress the individual fighting effort of each group on the total fighting efforts of its degree-one allies and enemies, respectively. Since these are endogenous and subject to a reflection problem which is standard in regressions involving networks (see Bramoullé, Djebbari and Fortin 2009; Liu *et al.* 2011), we use an IV strategy similar to that used by Acemoglu, Garcia-Jimeno and Robinson (2014). In particular, our identification exploits the exogenous variation in the average weather conditions facing, respectively, the set of allies and of enemies of each group. Our focus on weather shocks is motivated by the recent literature documenting that these have important effects on fighting intensity (see Dell 2012, Hidalgo *et al.* 2010, Jia 2014, Miguel, Satyanath and Sergenti 2004, and Vanden Eynde 2011). Without imposing any restriction on the estimation procedure, we find that the two estimated externalities have the (opposite) sign pattern, which aligns with the predictions of the theory.

After estimating the size of the network externalities, we perform two sets of policy-oriented counterfactual experiments. First, we remove sequentially each of the groups in conflict while letting all surviving group re-optimize their fighting effort. This key-player analysis is a policy-relevant exercise, as it can help international authorities to single out armed groups whose decommissioning would be most effective for scaling down a conflict. Interestingly, we find that while on average

[3] More details about the historical context of this conflict are provided in Section 3.1.

more active groups have a higher rank in the key player analysis, the relationship is far from one-to-one. For instance, the two factions of the Rally for Congolese Democracy (RCD) – Goma and Kisangani factions – rank, respectively, first and third in terms of their involvement in war episodes. Yet, their hypothetical removal would not contribute much to scaling down the conflict, because of their position in the network. According to our estimates, removing RCD-K would actually increase total violence, as the muted fighting of this group would be more than offset by the average increase in the effort of other armed groups. In contrast, the removal of the Hutu rebel group denominated Democratic Forces for the Liberation of Rwanda (FDLR) would yield a 14% reduction in violence. Our analysis also highlights the prominent role of foreign actors: eight out of the twenty groups bearing the largest responsibility for the escalation of violence are foreign national armies. Removing all foreign troops would reduce total violence by 24%, a large effect.

We also study a policy aimed at pacifying armed groups without removal, i.e., rewiring enmity links into neutral or alliance relationships. First, we show that pacifying all enmities in the Democratic Republic of Congo (henceforth, DRC) would yield a reduction in fighting by between 54% and 91%. Next, we show that a more realistic policy aiming to pacify individual groups could also be effective. For instance, rewiring all enmities of the FDLR into neutral relationships would yield a de-escalation of the conflict by 13%.

Our contribution is related to various strands of the existing literature. First, our paper is linked to the growing literature on the economics of networks (e.g., Acemoglu and Ozdaglar 2011, Bramoullé, Kranton, and d'Amours 2014, Jackson 2008, Jackson and Zenou 2014). There exist only very few papers in the literature studying strategic interactions of multiple agents in networks of conflict. Franke and Öztürk (2009) study agents being embedded in a network of bilateral conflicts, where agents can choose their fighting efforts to attack their neighbors. However, they do not allow for alliances. Moreover, they only characterize equilibrium efforts for particular networks (regular, star-shaped, and complete bipartite graphs), while we provide an equilibrium characterization for any network structure. Huremovic (2014) extends their model introducing endogenous fighting efforts in contests. His model differs from ours in several respects: there are no alliances, and the conflict is restricted to a collection of pairwise contests (while in our model many agents compete over a common prize). Neither of these studies tests the model empirically.

Two recent theoretical papers study the endogenous formation of networks in models of conflict. Hiller (2012) considers a model where agents can form alliances to coerce payoffs from enemies with fewer friends. Jackson and Nei (2014) study the formation of alliances in multilateral interstate wars and the implications on trade relationships between them, showing that trade can have a mitigating effect on conflicts. Neither of these papers consider, as we do, an endogenous choice of fighting effort. Different from these papers, we do not analyze the determinants of the network, which we infer from the data and treat as exogenous. Extending the analysis of endogenous network formation to large-scale networks including both enmities and alliances like the one in the Congo War is a challenging endeavor that is beyond the scope of this research.[4]

All of the above papers provide theoretical contributions. Our paper provides an estimation of the model based on the structural equations of the theory, and uses the estimates of the structural parameters to perform a key player analysis. In this sense, our paper is related to recent work by Acemoglu, Garcia-Jimeno and Robinson (2014) who estimate a political economy model of public

[4]From an empirical standpoint, most group characteristics that determine whether each two groups are friends, enemies or neutral are unobservable to the econometrician. The challenge is to predict how such shocks would reshuffle network links. We conjecture that the endogenous reshuffling of alliances and enmities in response to weather shocks – which is the source of identification in our model – must be rare. Consistent with this conjecture, we see only very little reshuffling of alliances during the Second Congo War. Thus, we do not view network endogeneity as a major threat to our econometric strategy.

goods provision using a network of Colombian municipalities. Their empirical strategy is related to ours, although they use historical variations in players' characteristics while we use panel variations (exogenous shocks in rainfall).

The key player analysis in strategic games with networks is pioneered by Ballester, Calvo-Armengol and Zenou (2006). The authors determine equilibrium effort choices in a game of strategic complements between neighboring nodes, and identify key players, i.e. the agents whose removal reduces equilibrium aggregate effort the most. However, their payoff structure is substantially different from ours, and does not incorporate an environment in which agents compete for common resources. More recently, the key player policy has been studied empirically. Liu *et al.* (2011) test the key player policy for juvenile crime in the United States, while Lindquist and Zenou (2013) identify key players for co-offending networks in Sweden.

Further, our study is broadly related to the growing politico-economic conflict literature. Papers in this literature typically focus on settings with two large groups facing each other, without considering network relationships. A few papers consider multiple groups comprising each a large number of players, and study collective action problems. Esteban and Ray (2001) show that the Olson paradox does not generally hold and that sometimes large groups can be more effective than small groups. To this purpose they build a model with n different groups composed each of a varying number of individual players. Their model is different and complementary to ours: they have no network structure, but focus on a setting comprising n different cliques, where there are no links between cliques. Related to that, Rohner (2011) constructs a conflict model allowing for n aggregate ethnic groups comprising a given number of individual players.

Our paper also shares some commonalities with the literature on the role of alliances in settings with either three players, or with identical players (see the surveys in Bloch 2012, and Konrad 2009 and 2011). Some papers note that alliances may be socially desirable since they reduce rent dissipation in wars (e.g. Olson and Zeckhauser 1966). Other papers regard the mere existence of alliances as a "puzzle" given the free-riding (collective action) problem they generate, and the fact that victorious coalitions must share resources when victorious (see Esteban and Sakovics 2004, Konrad and Kovenock 2009, and Nitzan 1991).

Finally, our paper is related to the empirical literature on civil war, and in particular to the recent literature that studies conflict using very disaggregated micro-data on geo-localised fighting events, such as for example Dube and Vargas (2013), Cassar, Grosjean, and Whitt (2013), La Ferrara and Harari (2012), Michalopoulos and Papaioannou (2013), Rohner, Thoenig, and Zilibotti (2013b). In a recent interesting paper on the conflict in the DRC, Sanchez de la Sierra (2014) studies how price shocks of particular metals (cobalt, gold) affect the incentives of armed groups to establish control of resource-producing villages in Eastern Congo.

The paper is organized as follows: Section 2 presents the theoretical model and characterizes the equilibrium; Section 3 discusses the application to the Second Congo War and presents the main estimation results. Section 4 performs a number of robustness checks, while Section 5 performs some policy counterfactual analyses (key player and pacification policies). Section 6 concludes. An appendix contains some technical material.

2 Theory

2.1 Environment

We consider a population of $n \in \mathbb{N}$ agents (armed groups) whose interactions are captured by a network $G \in \mathcal{G}^n$, where $\mathcal{G}^n$ denotes the class of graphs on n nodes (vertices). Each pair of agents can be in one of three states: alliance, enmity, or neutrality. We represent the set of bilateral states by the matrix $\mathbf{A} = (a_{ij})_{1\leq i,j\leq n}$ where $a_{ij} \in \{-1, 0, 1\}$. More formally, $\mathbf{A}$ is the signed adjacency matrix associated with the network G, where, for all $i \neq j$,

$$a_{ij} = \begin{cases} 1, & \text{if } i \text{ and } j \text{ are allies,} \\ -1, & \text{if } i \text{ and } j \text{ are enemies,} \\ 0, & \text{if } i \text{ and } j \text{ are in a neutral relationship.} \end{cases}$$

Note that a neutral relationship is modeled as the absence of links.

Let $a_{ij}^+ \equiv \max\{a_{ij}, 0\}$ and $a_{ij}^- \equiv -\min\{a_{ij}, 0\}$ denote the positive and negative parts of a_{ij}, respectively. Then, $a_{ij} = a_{ij}^+ - a_{ij}^-$, respectively, for all $1 \leq i, j \leq n$. Similarly, $\mathbf{A} = \mathbf{A}^+ - \mathbf{A}^-$ where $\mathbf{A}^+ = (a_{ij}^+)_{1\leq i,j\leq n}$ and $\mathbf{A}^- = (a_{ij}^-)_{1\leq i,j\leq n}$. We denote the corresponding subgraphs as G^+ and G^-, respectively, so that G can be written as the graph join $G = G^+ \oplus G^-$. Finally, we define by $d_i^+ \equiv \sum_{j=1}^n a_{ij}^+$ and $d_i^- \equiv \sum_{j=1}^n a_{ij}^-$, respectively, the number of allies and enemies of agent i.

The n agents compete for a prize whose total value is denoted by $V > 0$. We assume agents' payoffs to be determined by a generalized Tullock *contest success function* (CSF) (cf. Tullock 1980, and Skaperdas 1996). The CSF maps the relative fighting intensity each agent devotes to a conflict into the share of the prize he appropriates after the conflict. More formally, we postulate a payoff function $\pi_i : \mathcal{G}^n \times \mathbb{R}^n \to \mathbb{R}$ given by

$$\pi_i(G, \mathbf{x}) = \zeta_i + V \frac{\varphi_i(G, \mathbf{x})}{\sum_{j=1}^n \varphi_j(G, \mathbf{x})} - x_i \tag{1}$$

where $\zeta_i \geq 0$ is an exogenous endowment, $\mathbf{x} \in \mathcal{S}$ is a vector describing the fighting effort of each agent, and $\varphi_i \in \mathbb{R}$ is agent i's *operational performance* (OP). The latter is assumed to depend on agent i's own fighting effort, as well as on his allies' and enemies' efforts. More formally, we assume that

$$\varphi_i(G, \mathbf{x}) \equiv x_i + \beta \sum_{j=1}^n a_{ij}^+ x_j - \gamma \sum_{j=1}^n a_{ij}^- x_j, \tag{2}$$

where $\beta, \gamma \in [0, 1]$ are spillover parameters from allies' and enemies' fighting efforts, respectively. Note that the specification of equation (2) implies no heterogeneity across agents other than their position (i.e., the number of allies and enemies) in the network. We introduce other sources of heterogeneity (e.g., military power) in Section 2.6 below.

Equation (2) postulates that each agent's OP increases in the total effort exerted by its allies and decreases in the total effort exerted by its enemies. These externalities compound with those already embedded in standard CSFs, which equation (2) nests as a particular case when $a_{ij}^+ = a_{ij}^- = 0$ for all i and j. In this case, $\pi_i(G, \mathbf{x}) = \left(x_i / \sum_{j=1}^n x_j\right) V - x_i$, and each agent's effort imposes a negative externality on the other agents in the contest only by increasing the denominator of the CSF.

Consider, for example, a network such that $a_{kk'}^+ = 1$ and $\beta > 0$, for one and only one pair of agents (k, k') (while $a_{ij}^- = 0$ for all $i, j = 1, \ldots, n$). Then, $\pi_k(G, \mathbf{x}) = V(x_k + \beta x_{k'}) / (\sum_{i=1}^n x_i + \beta(x_k + x_{k'})) - x_k$. In this case, an increase in the effort of k' affects the payoff of k via two channels: (i) the standard negative externality working through the denominator; (ii) the positive externality working through the numerator. Thus, holding efforts constant, an alliance between two agents increases the share of V jointly accruing to them, at the expenses of the remaining groups. To the opposite, enmity links strengthen the negative externality of the standard CSF.

Note that the OP could in principle be negative (for instance, when an agent has many enemies exerting high effort). We interpret a negative OP as a situation in which a group is expropriated of part of its endowment (ζ_i) – since utility is linear in income, this entails no loss of generality.

In the Nash equilibrium characterized in Section 2.2 below, however, all groups' OP is positive.[5] Note further that there is no "participation decision", namely, belligerent groups cannot decide to abstain from the conflict and live from their endowment. This has a natural interpretation: groups cannot flee from the country, and not making any fighting effort would expose them to other armed groups' looting and ransacking activity. For technical reasons, we impose a lower bound to the choice set, i.e., $\mathbf{x} \in \mathcal{S} = [\underline{x}_1, \infty) \times [\underline{x}_2, \infty) \times ...[\underline{x}_n, \infty)$.

2.2 Equilibrium Fighting Effort

In this section, we characterize the Nash equilibrium of the contest. More formally, each agent chooses effort (x_i) non-cooperatively so as to maximize $\pi_i(G, [x_i, \mathbf{x}_{-i}])$, given $\mathbf{x}_{-i}$. The equilibrium is a fixed point of the effort vector.

Using equations (1)-(2) yields the following system of first order conditions (FOC), for $i = 1, 2, \ldots, n$:

$$\frac{\partial \pi_i(G, \mathbf{x})}{\partial x_i} = 0 \Longleftrightarrow \varphi_i = \frac{1}{1 + \beta d_i^+ - \gamma d_i^-} \left(1 - \frac{1}{V} \sum_{j=1}^{n} \varphi_j\right) \sum_{j=1}^{n} \varphi_j.$$

We assume that, for all i:

$$\beta d_i^+ - \gamma d_i^- > -1. \tag{3}$$

This condition is both necessary and sufficient for the second order condition to hold for all players at the equilibrium (see Proposition 1 and its proof in Appendix A). In the empirical analysis, we check that this condition holds for our estimates of β and γ.

Rearranging terms allows us to obtain a simple expression for the equilibrium OP level,

$$\varphi_i^*(G) = \Lambda^{\beta,\gamma}(G) \left(1 - \Lambda^{\beta,\gamma}(G)\right) \Gamma_i^{\beta,\gamma}(G) \times V, \tag{4}$$

and for the share of the prize accruing to i in equilibrium,

$$\frac{\varphi_i^*(G)}{\sum_{j=1}^{n} \varphi_j^*(G)} = \frac{\Gamma_i^{\beta,\gamma}(G)}{\sum_{j=1}^{n} \Gamma_j^{\beta,\gamma}(G)}, \tag{5}$$

where

$$\Gamma_i^{\beta,\gamma}(G) \equiv \frac{1}{1 + \beta d_i^+ - \gamma d_i^-}, \text{ and } \Lambda^{\beta,\gamma}(G) \equiv 1 - \frac{1}{\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G)}. \tag{6}$$

$\Gamma_i^{\beta,\gamma}(G)$ is a measure of the local hostility level capturing the externalities associated with agent i's first-degree alliance and enmity links. Both $\Gamma_i^{\beta,\gamma}(G)$ and $\Lambda^{\beta,\gamma}(G)$ are decreasing with β, and increasing with γ (see Lemma 3 in Online Appendix B). Note also that equation (1) implies that the share of the prize accruing to agent i increases in the number of her allies and decreases in the number of her enemies.

Next, we characterize the equilibrium fighting effort, and show how it depends on the structure of the network.

[5] Nor do we impose any non-negativity constraint on x_i, given the linearity of the pay-off function. The zero effort level is a matter of normalization. One could rewrite the model by replacing the effort level x_i with $(\bar{x} + x_i)$. All results would be unchanged.

Proposition 1. *Assume that* $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), d^-_{\max}\}$*, where* $\lambda_{\max}(\mathbf{A}^{\pm})$ *denotes the largest eigenvalue associated with the matrix* $\mathbf{A}^{\pm}$*. Assume, in addition, that condition (3) holds true. Let* $\Gamma_i^{\beta,\gamma}(G)$ *and* $\Lambda^{\beta,\gamma}(G)$ *be defined as in equation (6), and let*

$$\mathbf{c}^{\beta,\gamma}(G) \equiv \left(\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-\right)^{-1}\mathbf{\Gamma}^{\beta,\gamma}(G) \tag{7}$$

be a centrality vector, whose generic element $c_i^{\beta,\gamma}(G)$ *describes the centrality of agent* i *in the network* G*. Then, under an appropriate restriction of the strategy space,* $\mathcal{S} = [\underline{x}_1, \infty) \times [\underline{x}_2, \infty) \times \ldots \times [\underline{x}_n, \infty) \subset \mathbb{R}^n$*, where* $\underline{x}_i > -\infty\ \forall i = 1, 2, \ldots, n$*, there exists a unique interior Nash equilibrium of the* n*–player simultaneous move game with payoffs given by equation (1), agents' OPs in equation (2), where the equilibrium effort levels are characterized by*

$$x_i^*(G) = V\Lambda^{\beta,\gamma}(G)\left(1 - \Lambda^{\beta,\gamma}(G)\right)c_i^{\beta,\gamma}(G) \tag{8}$$

for all $i = 1, \ldots, n$*, where* $x_i^*(G) > \underline{x}_i$*. Moreover, the equilibrium OP levels are given by equation (4), and the equilibrium payoffs are given by*

$$\pi_i^*(G) = \pi_i(\mathbf{x}^*, G) = V(1 - \Lambda^{\beta,\gamma}(G))\left(\Gamma_i^{\beta,\gamma}(G) - \Lambda^{\beta,\gamma}(G)\,c_i^{\beta,\gamma}(G)\right). \tag{9}$$

The inequality $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), d^-_{\max}\}$ is a sufficient condition for the matrix in (7) to be invertible. Equation (9) follows from the set of FOCs. Condition (3) ensures that, for all $i = 1, \ldots, n$, agent i's payoff, conditional on $\mathbf{x}^*_{-i}$, is locally concave, implying that the FOCs pin down a local maximum for all players.[6] Formally, this guarantees that the strategy profile satisfying (9) is a *local* Nash equilibrium (Alos-Ferrer and Ania 2001).[7] To ensure that this strategy profile is, in addition, a Nash equilibrium in the standard sense, we must impose a lower bound on the state space to ensure in turn that, conditional on $\mathbf{x}^*_{-i}$, $x_i^*(G)$ is a best reply over the entire admissible region, $[\underline{x}_i, \infty)$.[8]

The centrality measure, $c_i^{\beta,\gamma}(G)$, plays a key role in Proposition 1. Note, in particular, that the relative fighting efforts of any two agents equals the ratio between the respective centrality in the network:

$$\frac{x_i^*(G)}{x_j^*(G)} = \frac{c_i^{\beta,\gamma}(G)}{c_j^{\beta,\gamma}(G)}.$$

2.3 The Case of Small Externalities

While $c_i^{\beta,\gamma}(G)$ depends, in general, on the entire network structure, it is instructive to consider networks in which the spillover parameters β and γ are small. In this case, our centrality measure can be approximated by the the sum of (i) the Bonacich centrality related to the network of enmities, G^-, (ii) the (negative-parameter) Bonacich centrality related to the network of alliances, G^+, and (iii) the local hostility vector, $\Gamma^{\beta,\gamma}(G)$.[9]

[6] In the rest of this section, we assume that the two conditions under which the Proposition is proven are satisfied. In the empirical analysis, we will check that the estimates of β and γ and the adjacency matrix ensure that the FOCs characterize an interior Nash equilibrium.

[7] A *local Nash equilibrium* is a strategy profile such that no player has an incentive to operate a "small" deviation. A formal definition is provided in Definition 2 in Appendix A. The lower bound on the state space ensures that, at the equilibrium, no player has an incentive to operate *any* feasible deviation.

[8] In Remark 1 in Appendix A we show that it is possible to compute a common lower bound in Proposition 1, $\underline{x}$, on the effort levels x_i, such that $\pi_i(\mathbf{x}^*_{-i}, x_i, G) - \pi_i(\mathbf{x}^*, G) < 0$ holds for all $i = 1, \ldots, n$ if $\mathbf{x} \in [\underline{x}, \infty)^n$ and that $\underline{x} < x_i^*$ for all $i = 1, \ldots, n$.

[9] See the Online Appendix D for a more detailed discussion of the Bonacich centrality. A discussion of the Bonacich centrality with a negative parameter can be found in Bonacich (2007) .

Lemma 1. *Assume that the conditions of Proposition 1 are satisfied. Then, as $\beta \to 0$ and $\gamma \to 0$, the centrality measure defined in equation (7) can be written as*

$$\mathbf{c}^{\beta,\gamma}(G) = \mathbf{b}_{\cdot}(\gamma, G^{-}) + \mathbf{b}_{\cdot}(-\beta, G^{+}) - \mathbf{\Gamma}^{\beta,\gamma}(G) + O\left(\beta\gamma\right),$$

where $O\left(\beta\gamma\right)$ involves second and higher order terms, and the ($\boldsymbol{\mu}$-weighted) Bonacich centrality with parameter α is defined as $\mathbf{b}_{\cdot}(\alpha, G) \equiv \mathbf{b}_{\mathbf{\Gamma}^{\beta,\gamma}(G)}(\alpha, G) = (\mathbf{I}_n - \alpha\mathbf{A})^{-1}\,\mathbf{\Gamma}^{\beta,\gamma}(G) = \sum_{k=0}^{\infty} \alpha^k \mathbf{A}^k \mathbf{\Gamma}^{\beta,\gamma}(G)$.

Lemma 1 states that the centrality $\mathbf{c}^{\beta,\gamma}(G)$ can be expressed as a linear combination of the weighted Bonacich centralities $\mathbf{b}_{\cdot}(\gamma, G^{-})$, $\mathbf{b}_{\cdot}(-\beta, G^{+})$ and the vector $\mathbf{\Gamma}^{\beta,\gamma}(G)$. Each Bonacich centrality gauges the network multiplier effect attached to the system of enmities and alliances, respectively. In particular, $\mathbf{b}_{\cdot}(\gamma, G^{-})$ captures how a group i is influenced by all its (direct and indirect) enemies.[10] In the case of of weak network externalities (i.e. when $\beta \to 0$ and $\gamma \to 0$), the Bonacich centrality can be itself approximated as follows:

$$\begin{aligned}
b_{\cdot,i}\left(\gamma, G^{-}\right) &= \Gamma_i^{\beta,\gamma}(G) + \gamma \sum_{j=1}^{n} a_{ij}^{-} \Gamma_j^{\beta,\gamma}(G) + \gamma^2 \sum_{j=1}^{n} a_{ij}^{-} \sum_{k=1}^{n} a_{jk}^{-} \Gamma_k^{\beta,\gamma}(G) + O\left(\gamma^3\right), \\
b_{\cdot,i}(-\beta, G^{+}) &= \Gamma_i^{\beta,\gamma}(G) + (-\beta) \sum_{j=1}^{n} a_{ij}^{+} \Gamma_j^{\beta,\gamma}(G) + (-\beta)^2 \sum_{j=1}^{n} a_{ij}^{+} \sum_{k=1}^{n} a_{jk}^{+} \Gamma_k^{\beta,\gamma}(G) + O\left(\beta^3\right).
\end{aligned}$$

Thus, Lemma 1 suggests that, when higher order terms can be neglected, our centrality measure is increasing in γ and in the number of first order and second order enmities, whereas it is decreasing in β and in the number of first order alliances. Second order alliances have instead a positive effect on the centrality measure.[11]

Moreover, we can also obtain a simple approximate expression for the equilibrium efforts and the payoffs in Proposition 1.[12]

Lemma 2. *As $\beta \to 0$ and $\gamma \to 0$, the equilibrium effort and payoff of agent i in network G can be written as*

$$\begin{aligned}
x_i^*(G) &= V\left(A^{\beta,\gamma}(G) - B\left(\beta d_i^{+} - \gamma d_i^{-}\right)\right) + O\left(\beta\gamma\right), \\
\pi_i^*(G) &= V\left(C^{\beta,\gamma}(G) + D\left(\beta d_i^{+} - \gamma d_i^{-}\right)\right) + O\left(\beta\gamma\right),
\end{aligned}$$

where $A^{\beta,\gamma}(G), B, C^{\beta,\gamma}(G)$ and D are positive constants with $A^{\beta,\gamma}(G)$ and $C^{\beta,\gamma}(G)$ being of the order of $O(\beta) + O(\gamma)$.

[10] The Bonacich centrality measure related to the network of hostilities, $b_{\cdot,i}\left(\gamma, G^{-}\right)$, measures as the local hostility levels along all walks reaching i using only hostility connections, where walks of length k are weighted by the geometrically decaying hostility externality γ^k (see also the Online Appendix D). Due to the approximation, we only consider links up to order two.

[11] The intuition for this property is as follows. If i is allied with j and j is allied with k, an increase in the fighting effort of k reduces the fighting effort of j and this, in turn, increases the fighting effort of i. Consider, instead, the case in which i is an enemy of j and j is an enemy of k. Then, an increase in the fighting effort of k increases the fighting effort of j and this, in turn, increases the fighting effort of i.

[12] See the proof of Lemma 2 in Appendix A for the explicit expressions for the constants $A^{\beta,\gamma}(G), B, C^{\beta,\gamma}(G)$ and D.

It is useful to note that, when $\beta = \gamma = 0$, then $\Lambda^{\beta,\gamma}(G) = 1 - \frac{1}{n}$, and $c_i^{\beta,\gamma}(G) = 1$. Then, the equilibrium expressions in Proposition 1 simplify to $x_i^*(G) = V(n-1)/n^2$ and $\pi_i^*(G) = V/n^2$ which are the standard solutions in the Tullock CSF.

Lemma 2 shows that, when network externalities are small, an agent's fighting effort increases in the weighted difference between the number of enmities (weighted by γ) and alliances (weighted by β), i.e. the *net local externalities* $d_i^+\beta - d_i^-\gamma$. The opposite is true for the equilibrium payoff, which is increasing in $d_i^+\beta - d_i^-\gamma$. Thus, *ceteris paribus*, an increase in the spillover from alliances (enmities), parameterized by β (γ), and an increase in the number of allies (enemies) decreases (increases) agent i's fighting effort and increases (reduces) its payoff. Intuitively, an agent with many enemies tends to fight harder and to appropriate a smaller share of the prize, whereas an agent with many friends tends to fight less and to appropriate a large size of the prize.

One must remember, however, that this simple result may fail to hold true if β and γ are not small and higher-degree links have sizeable effects. In the empirical analysis below, we find that higher order terms cannot be ignored to draw both positive and normative implications.

2.4 Rent Dissipation and Key Player

In this section, we discuss normative implications of the theory. Our welfare measure is (minus) the total *rent dissipation*. Since V is exogenous, the rent dissipation equals the total equilibrium fighting effort:

$$\mathrm{RD}^{\beta,\gamma}(G) \equiv \frac{1}{V}\sum_{i=1}^{n} x_i^*(G) = \Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))\sum_{i=1}^{n} c_i^{\beta,\gamma}(G).$$

Since $\sum_{i=1}^{n}\pi_i^*(G) = V\left(1-\mathrm{RD}^{\beta,\gamma}(G)\right)$, minimizing rent dissipation is equivalent to maximizing welfare.

Next, we define the *key player* to be the agent whose removal triggers the largest reduction in rent dissipation (cf. Ballester, Calvo-Armengol and Zenou 2006).

Definition 1. *Let $G\backslash\{i\}$ be the network obtained from G by removing agent i, and assume that the conditions of Proposition 1 hold. Then the key player $i^* \in \overline{\mathcal{N}} \equiv \{1,\ldots,n\}\cup\emptyset$ is defined by*

$$i^* = \arg\max_{i\in\overline{\mathcal{N}}}\left\{\mathrm{RD}^{\beta,\gamma}(G) - \mathrm{RD}^{\beta,\gamma}(G\backslash\{i\})\right\}. \tag{10}$$

Note that the welfare difference $\mathrm{RD}^{\beta,\gamma}(G) - \mathrm{RD}^{\beta,\gamma}(G\backslash\{i\})$ can be interpreted as the maximum cost that a benevolent policy maker is willing to pay to induce (or force) agent i not to participate in the contest. In Proposition 2 in the Online Appendix B, we show that the identity of the key player is related to the centrality measure defined in equation (7).[13]

2.5 Examples

In this section, we discuss two types of examples that illustrate general properties of the model. We consider, first, a regular graph where all agents have a symmetric position in the network. This graph is useful for showing how the number of alliances and enmities affects rent dissipation. Then, we consider a path graph with a core-periphery structure that highlights the importance of the centrality of agents in the network.

[13] The key player identified in equation (43) in Proposition 2 in the Online Appendix B differs significantly from the one introduced in Ballester, Calvo-Armengol and Zenou (2006). Our key player formula is more involved due to the non-linearity inherent in the contest success function in the agents' payoffs.

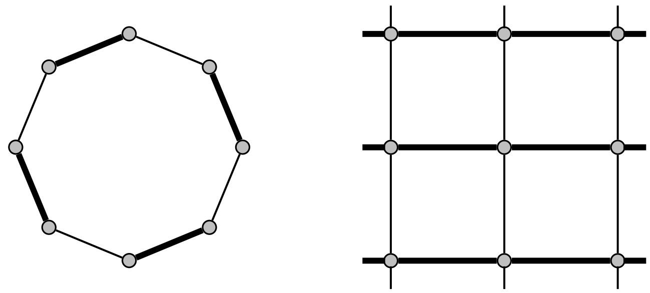


Figure 1: The figure shows three examples of regular graphs G_{k^+,k^-}: The left panel shows a regular graph with $k^+ = k^- = 1$ (cycle), the middle panel shows a regular graph with $k^+ = k^- = 2$ (a lattice with periodic boundary conditions, i.e. a torus), and the right panel a Cayley graph with $k^+ = 1$ and $k^- = 2$. Alliances are indicated with thick lines while enmities are indicated with thin lines.

2.5.1 Regular Graph

A regular network, G_{k^+,k^-}, has the property that every agent i has $d_i^+ = k^+$ alliances and $d_i^- = k^-$ enmities. Regular graphs are tractable and enable us to perform comparative statics on the number of alliances or enmities in the network. Three examples of regular graphs are displayed in Figure 1.

Given the symmetric structure, there exists a symmetric Nash equilibrium such that all agents exercise the same effort. Moreover, $\varphi_i^* = \varphi^* = 1/n$ for all $i = 1, \ldots, n$, implying an equal division of the pie. Under the conditions of Proposition 1, the equilibrium effort and payoff vectors are given, respectively, by:

$$x^*\left(k^+, k^-\right) \equiv x_i^*\left(G_{k^+,k^-}\right) = \left(\frac{1}{1+\beta k^+ - \gamma k^-} - \frac{1}{n}\right) \times \frac{V}{n}, \tag{11}$$

$$\pi^*\left(k^+, k^-\right) \equiv \pi_i^*\left(G_{k^+,k^-}\right) = \frac{1+(1+n)(\beta k^+ - \gamma k^-)}{n(1+\beta k^+ - \gamma k^-)} \times \frac{V}{n}. \tag{12}$$

Standard differentiation implies that x^* is decreasing in k^+ and increasing in k^-, whereas π^* is increasing in k^+ and decreasing in k^-. Intuitively, alliances (enmities) reduce (increase) effort and rent dissipation by decreasing (increasing) the marginal return of individual fighting effort. This basic intuition gets confounded in general networks due to the asymmetries in higher-order links. However, the result is unambiguous in regular graphs.

The regular graph nests three interesting particular cases. First, if $\beta = \gamma = 0$, we have a standard Tullock game, with $\mathrm{RD}^{0,0}\left(G_{k^+,k^-}\right) = (n-1)/n$. Second, consider a complete network of alliances ($k^+ = n-1$), where, in addition, $\beta \to 1$. Then, $x^* \to 0$ and $\mathrm{RD}^{1,\gamma}\left(G_{n-1,0}\right) \to 0$, i.e., there is no rent dissipation. Namely, the society attains peacefully the equal split of the surplus, as in Rousseau's harmonious society. The lack of conflict does not stem here from social preferences or cooperation, but from the equilibrium outcome of a non-cooperative game between selfish individuals. The crux is the strong fighting externality across allied agents, which takes the marginal product of individual fighting effort down to zero. Third, consider, conversely, a society in which all relationships are hostile, i.e., $k^- = n-1$. Then, $\mathrm{RD}^{\beta,\gamma}\left(G_{0,n-1}\right) \to 1$ as $\gamma \to 1/\left(n-1\right)^2$: all rents are dissipated through fierce fighting and total destruction, as in Hobbes' homo homini lupus pre-contractual society.

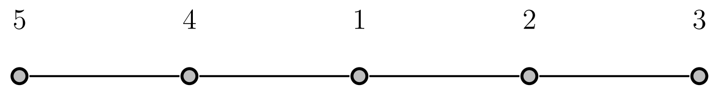


Figure 2: The figure shows a path graph, P_5, with five agents.

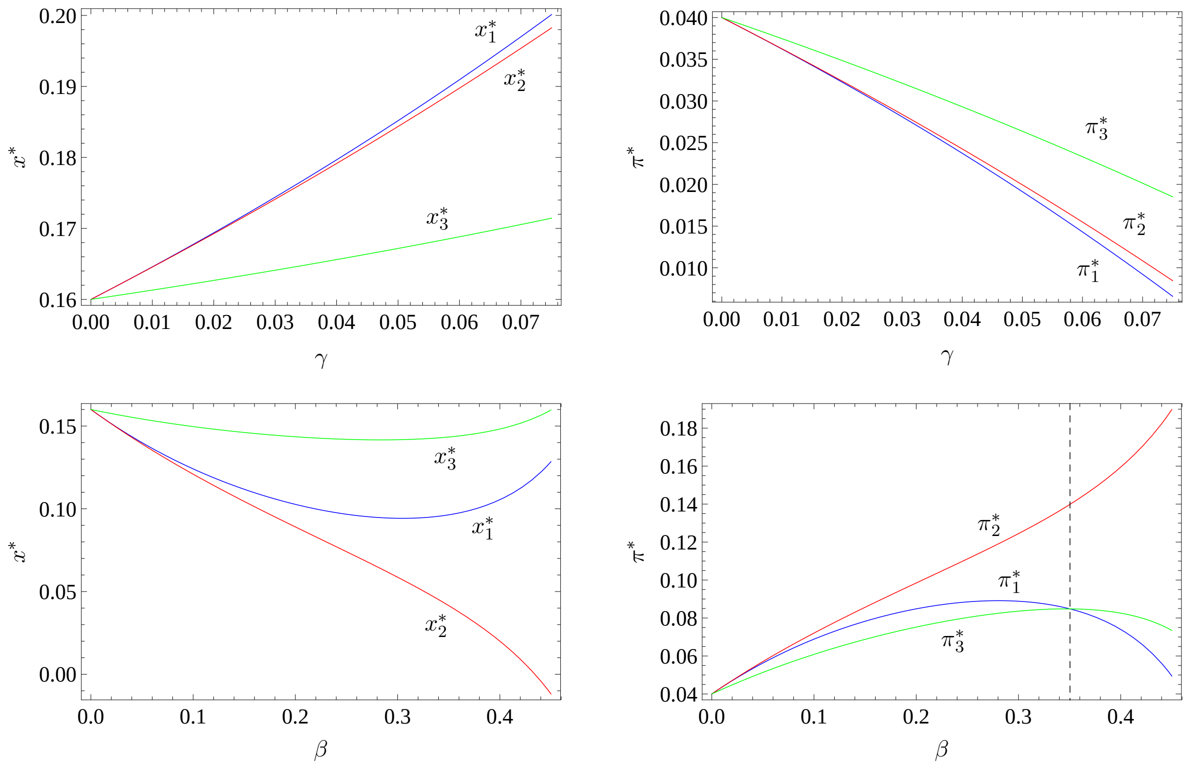


Figure 3: The figure shows the equilibrium efforts (left panels) and payoffs (right panels) as functions of γ and β for two path graphs in which there are only enmity links (upper panels) and only alliance links (lower panels), respectively, for $V = 1$.

2.5.2 Path Graph

Next, we consider a path (core-periphery) graph, P_5, with five agents, see Figure 2. No link means a neutral relationship. Suppose, first, that all links in Figure 2 are enmities. The upper left and upper right panel of Figure 3 show, respectively, effort levels and payoff for different values of γ. The ranking of the effort level yields $x_1^*(P_5) > x_2^*(P_5) = x_4^*(P_5) > x_3^*(P_5) = x_5^*(P_5)$.[14] Intuitively, effort is proportional to the centrality in the network. The ranking of pay-offs is opposite: welfare increases as one moves from the center to the periphery. Fighting effort and payoffs are increasing and decreasing in γ, respectively.

Consider, next, the polar opposite case in which all links in Figure 2 are alliances. The bottom left and right panels of Figure 3 show, respectively, effort levels and payoffs for different values of β.

[14] More formally, we have: $x_1^*(P_5) = V\frac{2(\gamma(\gamma+2)-2)\left(\gamma\left(3\gamma^2+\gamma-1\right)-1\right)}{(5-7\gamma)^2\left(3\gamma^2-1\right)}$, $x_2(P_5)^* = x_4^*(P_5) = V\frac{2\left((\gamma(3\gamma+5)-9)\gamma^2+2\right)}{(5-7\gamma)^2\left(1-3\gamma^2\right)}$ and $x_3^*(P_5) = x_5^*(P_5) = V\frac{2(\gamma(\gamma+2)-2)((\gamma-1)\gamma(3\gamma+1)+1)}{(5-7\gamma)^2\left(3\gamma^2-1\right)}$.

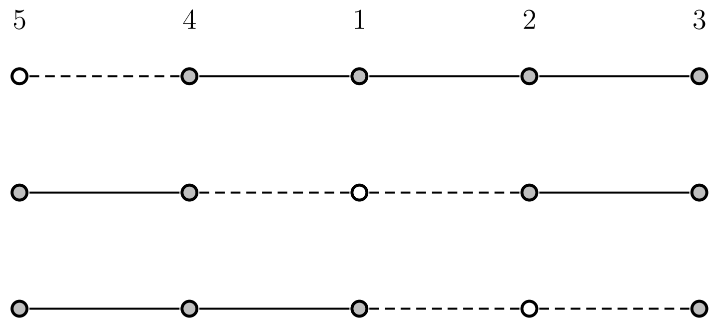


Figure 4: The figure shows three different graphs, P_4, $P_2 \cup P_2$ and $P_3 \cup P_1$, obtained from a path graph, P_5, due to the removal of an agent.

The ranking of effort yields now, $x_2^*(P_5) = x_4^*(P_5) < x_1^*(P_5) < x_3^*(P_5) = x_5^*(P_5)$.[15] For a low range of β's, the peripheral agents, 3 and 5, exert the highest effort and earn the lowest payoff. However, for higher β's, agents 3 and 5 earn a higher payoff than agent 1. Note that agents 2 and 4 exert the lowest effort, in spite of agent 1 being the most central player. The reason is that agents 2 and 4 are connected, respectively, to agents 3 and 5 who lie at the periphery of the network, and who have no other connections. Thus, agents 2 and 4 can free ride on the high effort exerted by 3 and 5.[16] Finally, the efforts and payoffs of agents 1, 3 and 5 are non-monotonic, due to the fact that, when β is high, there is more free riding. Agents 2 and 4 exert very low fighting effort, and induce substitution effects (i.e., higher effort) from their respective neighbors.

The results are consistent with Lemma 2. In the upper panel, as $\gamma \to 0$ the effort is lowest and payoff is highest for the agent who has the smallest number of enemies (i.e., agent 1). In the lower panel, as $\beta \to 0$ the effort is highest for the agents who have only one ally (i.e., agents 3 and 5).

Finally, we perform a key player analysis. Figure 4 shows the three different graphs, P_4, $P_2 \cup P_2$ and $P_3 \cup P_1$, obtained from removing one agent from P_5. When all links are enmities, the maximum reduction in rent dissipation is attained by removing the central agent, 1. The reason is that, in this case, a graph with more walks (hence, stronger network feedback effects) amplifies fighting. Removing the central agent in P_5 (as displayed in $P_2 \cup P_2$) yields the largest reduction in the number of walks, resulting in the lowest total fighting. In contrast, when all links are alliances, fighting is decreasing in the number of walks. In this case, the maximum reduction in rent dissipation is attained by removing a peripheral agent (either 3 or 5).

2.6 Heterogeneous Fighting Technologies

So far, we have maintained that all agents have access to the same technology turning fighting effort into OP. This was useful for keeping the focus sharply on the network structure. In reality, armed groups typically differ in size, wealth, access to weapons, etc. In this section, we generalize our model by allowing fighting technologies to differ across players. We restrict attention to additive heterogeneity, since this is crucial for achieving identification in the econometric model presented

[15] The analytical expressions are $x_1^*(P_5) = V\frac{2\left(\beta^2(\beta+1)(\beta(3\beta-10)+5)-2\right)}{(7\beta+5)^2\left(3\beta^2-1\right)}$, $x_2^*(P_5) = V\frac{2\left(\beta^2(\beta(5-3\beta)+9)-2\right)}{(7\beta+5)^2\left(3\beta^2-1\right)}$ and $x_3^*(P_5) = V\frac{2((\beta-2)\beta-2)(\beta(\beta+1)(3\beta-1)-1)}{(7\beta+5)^2\left(1-3\beta^2\right)}$.

[16] This is related to the interpretation of the Bonacich centrality with a negative parameter. In this case a node is more powerful to the degree that its connections themselves have few alternative connections (see Sec. 1.1.1 in Bonacich, 2007).

below.[17]

Suppose that agent i's OP is given by:

$$\varphi_i(G, \mathbf{x}) = \tilde{\varphi}_i + x_i + \beta \sum_{j=1}^{n} a_{ij}^{+} x_j - \gamma \sum_{j=1}^{n} a_{ij}^{-} x_j, \tag{13}$$

where $\tilde{\varphi}_i$ is a group-specific shifter affecting OP (e.g., its military capability).[18]

In the appendix, we show that the equilibrium OP is unchanged, and continues to be given by equation (4). Likewise, equation (5) continues to characterize the share of the prize appropriated by each agent. Somewhat surprising, the share of resources appropriated by group i, $\varphi_i^*(G)/\sum_{j=1}^{n} \varphi_j^*(G)$, is independent of $\tilde{\varphi}_i$. However, $\tilde{\varphi}_i$ affects the equilibrium effort exerted by group i and its payoff. In particular, the vector of the equilibrium fighting efforts is now given by

$$\mathbf{x}^* = (\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)^{-1} (V \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G)) \mathbf{\Gamma}^{\beta,\gamma}(G) - \tilde{\boldsymbol{\varphi}}), \tag{14}$$

where the definitions of $\Lambda^{\beta,\gamma}(G)$ and $\mathbf{\Gamma}^{\beta,\gamma}(G)$ are unchanged (see Proposition 3 in the Online Appendix B).

Equation (13) will be the basis of our econometric analysis where we introduce both observable and unobservable sources of heterogeneity. In particular, time-varying shocks to $\tilde{\varphi}$ will be key for the econometric identification.

3 Empirical Application - The Second Congo War

In this section, we apply the theory to the recent civil conflict in the Democratic Republic of Congo (henceforth, DRC). Our goal is to estimate the externality parameters β and γ from the structural equation (13) characterizing the Nash equilibrium of the model. Equipped with point estimates of the structural parameters we test key restrictions imposed by the theory and perform counterfactual pacification policies. We start by presenting the historical context of the DRC conflict. Then, we discuss the data sources and how the network structure is observed from the data. Finally, we proceed to the econometric model and to the discussion of our identification and estimation procedure.

3.1 Historical Context

We study the Second Congo War, sometimes referred to as the "Great African War". Detailed accounts of this conflict can be found in Prunier (2011) and Stearns (2011). The DRC is the largest Sub-Saharan African country in terms of area, and is populated by about 75 million inhabitants. It is a classic example of a failed state. After gaining independence from Belgium in 1960, it experienced recurrent political and military turbulence that turned it into one of the poorest countries in the world, despite its abundance of natural resources (including diamonds, copper, gold and cobalt). The DRC is also a very ethnically fragmented country with over 200 ethnic groups. The

[17] It is possible to solve for richer forms of heterogenity, such as a multiplicative one. However, we do not focus on such alternative cases, since it becomes impossible to identify econometrically the parameters of the model.

[18] An alternative interpretation is to consider our model as the linear approximation of a *logit-form* CSF (cf. Skaperdas, 1996). Consider the payoff function $\pi_i : \mathcal{G}^n \times \mathbb{R}^n \to \mathbb{R}$ with $\pi_i(G, \mathbf{x}) \equiv V \frac{e^{\lambda \varphi_i(G,\mathbf{x})}}{\sum_{j=1}^{n} e^{\lambda \varphi_j(G,\mathbf{x})}} - x_i$, for all $i = 1, \ldots, n$, where $\varphi_i(G, \mathbf{x})$ is the OP of agent i given by equation (2). In the limit $\lambda \to 0$ we obtain the ratio form $\pi_i(G, \mathbf{x}) = V \frac{1+\lambda\varphi_i(G,\mathbf{x})+O(\lambda^2)}{n+\lambda \sum_{j=1}^{n} \varphi_j(G,\mathbf{x})+O(\lambda^2)} - x_i = V \frac{\lambda^{-1}+\varphi_i(G,\mathbf{x})+O(\lambda)}{n\lambda^{-1}+\sum_{j=1}^{n} \varphi_j(G,\mathbf{x})+O(\lambda)} - x_i$. Hence, we can introduce the *shifted* OP from equation (13) with $\tilde{\varphi}_i = \lambda^{-1}$ for all $i = 1, \ldots, n$.

Congo conflict is emblematic of the role of natural resource rents and of the involvement of many inter-connected domestic and foreign actors. In particular, the conflict involved three Congolese rebel movements, 14 foreign armed groups, and several militias (Autesserre 2008). In such complex and fragmented warfare, alliances and enmities play a major role.

The Congo Wars are intertwined with the ethnic conflicts in neighboring Rwanda and Uganda. The culminating event is the Rwandan genocide of 1994, where the Hutu-dominated government of Rwanda supported by ethnic militias such as the Interahamwe, persecuted and killed nearly a million of Tutsis and moderate Hutus in less than one hundred days. After losing power to the Tutsi rebels of the Rwandan Patriotic Front (RPF), over a million Hutus fled Rwanda and found refuge in the DRC, governed at that time by the dictator Mobutu Sese Seko. The refugee camps hosted, along with civilians, former militiamen responsible of the Rwandan genocide. These continued to clash with the Tutsi population living both in Rwanda and in the DRC, most notably in the Kivu region (Seybolt 2000).

As ethnic tensions escalated, a broad coalition centered around Uganda and Rwanda, but also comprising several other African states, supported the anti-Mobutu rebellion led by Laurent-Désiré Kabila's Alliance of Democratic Forces for the Liberation of Congo (ADFL). The First Congo War (1996-97) ended with Kabila's victory. However, the relationship of the new president of the DRC with his former Tutsi allies and his main foreign sponsors, Rwanda and Uganda, deteriorated rapidly. The Second Congo War erupted in 1998 where Kabila received the support of some former foreign allies (Angola, Chad, Namibia, Sudan and Zimbabwe) and of the Hutu militias that had previously supported Mobutu.[19] His main enemies were Uganda, Rwanda and a network of rebel groups. Many rebel groups were linked to foreign powers: Uganda sponsored the Rally for Congolese Democracy - Kisangani (RCD-K), while the Congolese Liberation Movement (MLC), the Rally for Congolese Democracy - Goma (RCD-G) received support from Rwanda (Seybolt 2000). The heart of the war was the Eastern border and the conflict spread fast to the whole country, with many actors taking part in the conflict out of hostility to other specific actors. These include, among others, the anti-Ugandan rebel forces of the Allied Democratic Forces and the Lord's Resistance Army, or the anti-Angolan UNITA forces.

The Second Congo War ended officially in 2003, although fighting is still going on today. It is the deadliest conflict since World War II, with between 3 and 5 million lives lost (Olsson and Fors, 2004; Autesserre, 2008). In spite of the numerous shifts that occurred in 1997, the web of alliances and enmities remained remarkably stable thereafter. As documented by Prunier (2011: 187ff), most alliances and enmities were determined by international and domestic factors of the countries involved, and remained constant over the entire conflict (at least until 2010, the extent of time covered by our data). Many groups chose their loyalties following *Realpolitik* and trying to balance rising neighbors like South Africa or Sudan.

While many groups involved in the Second Congo War took stands *vis-a-vis* the DRC government, there were flagrant violations of transitivity, and the conflict cannot be described as a coherent two-camp (pro- versus anti-DRC government) war. In Prunier's words, "the continent was fractured, not only for or against Kabila, but *within* each of the two camps" (2011: 187). Recurrent clashes erupted among the main factions - like in the case of the infamous violent clashes between RCD-G and RCD-K (Prunier 2011: 240ff; Turner 2007: 200). Even the historical alliance between Uganda and Rwanda, which was forged on ethnic and geopolitical grounds cracked a few times, resulting in important clashes between the two armies or between groups sponsored by either of them (Turner, 2007: 200). Similarly, there was in-fighting among different pro-government paramilitary groups, such as the Mayi-Mayi militias, (cf. Prunier 2011: 281). The DRC army

[19] In 2001, Laurent-Désiré Kabila was assassinated, and was later replaced by his son Joseph Kabila.

itself was notoriously prone to internal clashes and mutinies, spurred by the fact that its units are segregated along ethnic lines and often correspond to former ethnic militias or paramilitaries that got integrated into the national army (cf. Prunier 2011: 305ff).[20] In summary, far from being a war between two unitary alliances, the conflict engaged a complex web of alliances and enmities, with many non-transitive links.

Two other aspects of the conflict are noteworthy. First, with the exception of the DRC armed forces, most actors were active in limited parts of the country. Ethnic militias (i.e., the vast majority of local armed groups) typically fought in territories adjacent to those where their affiliated population lived. Second, weather conditions were important in determining the intensity of the conflict in different regions. Figure 5 displays the fighting intensity and average climate conditions for different ethnic homelands in the DRC.[21] Weather conditions vary considerably both across regions of the DRC and over time.

3.2 Data

We build a panel dataset for the period 1998-2010 that includes both the official years of the Second Congo War (until 2003) and its turbulent aftermath. The unit of observation is at the fighting group×year level (annual frequency). The variables used in the estimations are built from a variety of data sources. We describe these and the construction of the variables in this section. The related summary statistics are displayed in Table 1.

Fighting – Our main data source is the Armed Conflict Location and Events Dataset (ACLED 2012).[22] This dataset contains 4765 geo-localised violent events taking place in the DRC involving 85 fighting groups. For each such event, ACLED provides information on the exact location, the date and the identities of the involved groups – including information about which groups fight on the same side and which fight on opposite sides.[23] In the recent literature ACLED has been used so far with the purpose of building geolocalized measurement of violence. Here, besides geolocalization, we also exploit bilateral information about which groups fight together or against each other to document the fine-grained structure of the network of alliances and enmities. To the best of our knowledge, our study is the first to exploit this information.

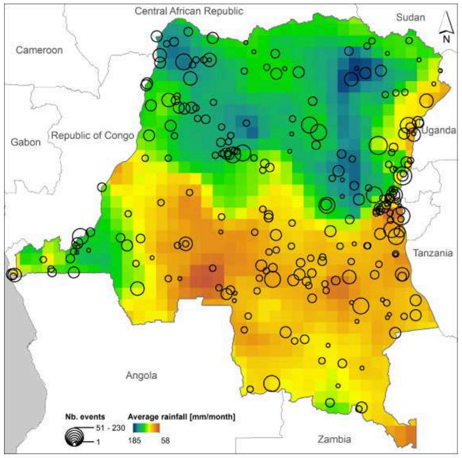
Our main dependent variable is group i's yearly *Fighting Effort*. We measure x_{it} as the sum over all ACLED fighting events involving group i in year t. In the robustness section, we construct

[20] Turner (2007) describes a typical clash between army fractions in 2004: "In the aftermath of the peace agreements of Sun City and Pretoria, Congo was supposed to create a unified national army and civil-territorial administration. Some Rwandophone officers of North and South Kivu led the resistance to *brassage* (intermingling) of officers and troops from various composants. The two most prominent of these were Colonel Jules Mutebutsi (a Munyamulenge from South Kivu) and General Laurent Nkunda (Rwandophone Tutsi allegedly from Rutshuru in Kivu). These officers led a mutiny against their superiors, and briefly took over the city of Bukavu (capital of South Kivu) (2007: 96)."

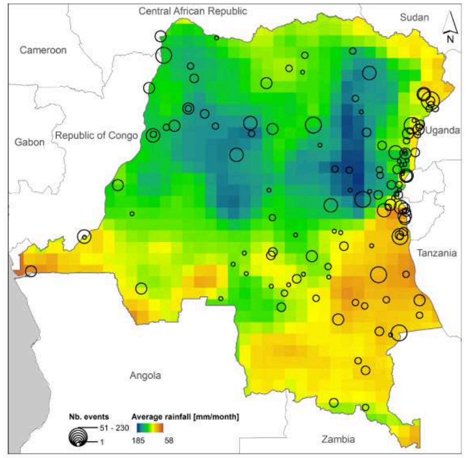
[21] The data used for generating this figure are discussed in detail below.

[22] This a well established data source in the literature. Recent papers using ACLED include, among others, Berman et al. (2014), Cassar, Grosjean, and Whitt (2013), Michalopoulos and Papaioannou (2013), and Rohner, Thoenig, and Zilibotti (2013b).

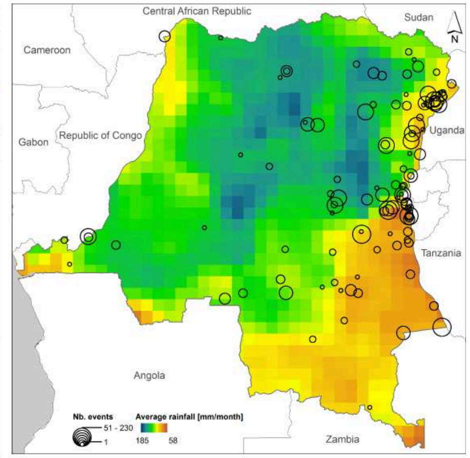
[23] Here are three examples: On the 18th of May 1999 a battle took place between "RCD: Rally for Congolese Democracy (Goma)" and "Military Forces of Rwanda", on one side, and the "Military Forces of DRC" on the other side. On the 13th of January 2000 there was a battle between "Lendu Ethnic Militia" and "Military Forces of DRC", on the one side, and "Hema Ethnic Militia" and "RCD: Rally for Congolese Democracy", on the other side. On the 3rd of February 2000, the "MLC: Congolese Liberation Movement" together with "Military Forces of Uganda" confronted the allied forces of the "Military Forces of DRC" and "Interahamwe Hutu Ethnic Militia".



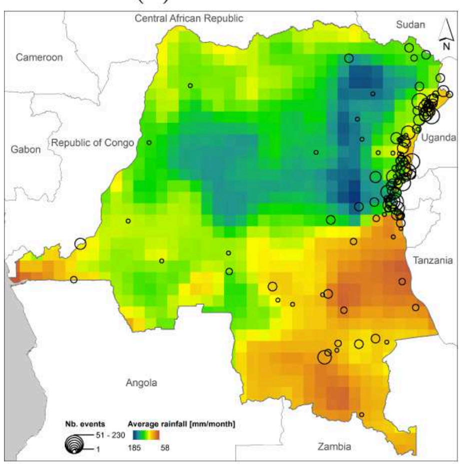
(a) 1998-1999



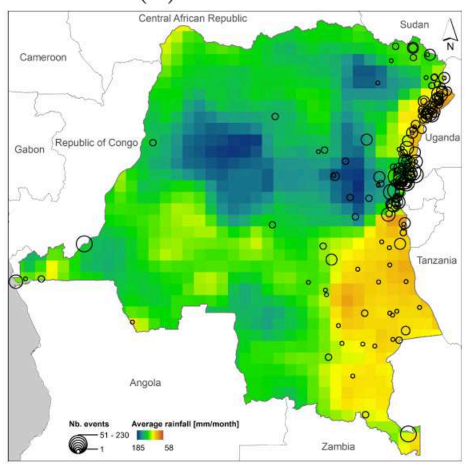
(b) 2000-2001



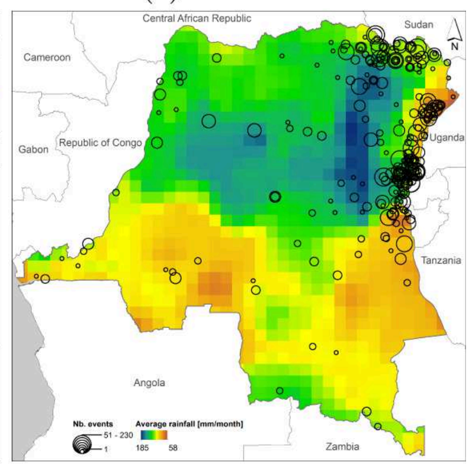
(c) 2002-2003



(d) 2004-2005



(e) 2006-2007



(f) 2008-2010

Figure 5: Average rainfall and number of violent events (ACLED) in the DRC.

Table 1: Summary statistics.

Variable	Obs.	Mean	Std. Dev.	Min.	Max.
Total Fighting	1190	5.23	22.72	0	300
Total Fighting of Enemies	1190	50.48	92.89	0	645
Total Fighting of Allies	1190	38.80	74.77	0	493
d^- (Number Enemies)	1190	2.61	3.59	0	20
d^+ (Number Allies)	1190	2.24	3.41	0	18
Foreign	1190	0.31	0.46	0	1
Government Organization	1190	0.20	0.40	0	1
Rain fall $(t-1)$	1190	125.35	26.36	58.02	197.35

alternative measures of fighting effort by restricting the count to the more conspicuous events such as those classified by ACLED as battles or those involving fatalities.

Rainfall – For the purpose of our instrumentation strategy we build the yearly average of rainfall in the homeland of each fighting group. We use gauge-based rainfall measure from the Global Precipitation Climatology Centre (GPCC) (Schneider *et al.* 2011), at a spatial resolution of $0.5° \times 0.5°$ grid-cells. This dataset is widely used and is renowned for its precision and fine resolution. The homeland of a fighting group corresponds to the spatial zone of its fighting operations (i.e. convex hull containing all geolocalized ACLED events involving that group at any time during the period 1998-2010). Then, for each year t, we compute the average rainfall in the grid-cell of the homeland centroid. Alternative constructions are considered in our robustness analysis.[24]

One potential concern is that rain-gauges located at the ground level may be damaged by local fighting activities. Therefore, in the robustness section, we also use satellite-based rainfall measures. These are generally less precise, but it is safe to assume that the measurement error is uncorrelated with ground-level fighting. We use two alternative annual dataset. The first comes from the Global Precipitation Climatology Project (GPCP) from NOAA and has a spatial resolution of $2.5° \times 2.5°$. The second is the Tropical Rainfall Measuring Mission (TRMM) from NASA at a resolution of $0.5° \times 0.5°$. These dataset use atmospheric parameters (e.g. cloud coverage, light intensity) as indirect and noisy measures of rainfall. Hence both of these satellite-based measures are much less precise than the rain-gauge measure.

Covariates – The following variables are used to generate the set of control variables. *Government Organization* is a binary variable equal to 1 for fighting groups that are officially affiliated to a domestic or foreign government. This amounts to 17 groups out of 85, corresponding to the military and police forces of Angola, Burundi, Chad, DRC, Namibia, Rwanda, South Africa, Sudan, Uganda, Zambia and Zimbabwe. The dummy *Foreign* is equal to 1 for all foreign actors, and 0 for all fighting groups that originate from the DRC. In total 26 groups are coded as Foreign. Finally for all groups we compute the yearly amount of fighting events in which they are involved outside the DRC. The resulting variable, *Fighting Effort Outside DRC*, is used as proxy for the global scope of operation of a group.

[24] One option consists in averaging rainfall across all grid-cells of the homeland (not only at the centroid). As for alternative construction of the homeland we consider an ellipsoid based on a subset of ACLED events (i.e. within one standard-deviation) or an homeland based on the ethnic affiliations of the fighting group. The main results are reported in the robustness section.

3.3 The Fighting Network

We estimate the network of alliances and enmities using two data sources. First, we use the Yearbook of the Stockholm International Peace Research Institute (SIPRI, see Seybolt, 2000). Second, we use the dyadic information provided by ACLED. Details are provided below.

As our primary criterion, we follow the classification provided by SIPRI, which lists alliances between the major actors, including both actors operating in the same region and groups fighting in different parts of the country but supporting each other logistically. The limitation of the SIPRI data is that they do not cover small armed groups and militias nor do they contain detailed information about bilateral enmity links.

For this reason, we also use the dyadic information provided by ACLED. In particular, we code two groups (i, j) as *allies* (i.e. $a_{ij}^+ = a_{ji}^+ = 1$) if they have been observed fighting on the same side in at least one occasion during the sample period, and if, in addition, they have never been observed fighting on opposite sides. Conversely, we code two groups as *enemies* (i.e. $a_{ij}^- = a_{ji}^- = 1$) if they have been observed fighting on opposite sides on at least two occasions, and they have never been observed fighting on the same side.[25] We code all other dyads as *neutral* (i.e. $a_{ij}^+ = a_{ij}^- = 0$).

Slightly less than 3% of the dyads are coded as allies, and slightly more than 3% are coded as enemies. The remaining dyads are classified as neutral, either because they were never involved jointly in any fighting event (93% of the dyads), or because of ambiguities, namely, the two groups were recorded fighting sometimes on the same side, and sometimes on opposite sides. The proportion of all dyads that are classified as neutral due to such an ambiguity is less than 1%. On average a group has 2.6 enemies and 2.2 allies (see Table 1).

Our methodology may raise three concerns. First, we assume a time-invariant network. One might fear that alliances and enmities get reshuffled throughout the war. From a historical perspective it appears as if many changes in the system of alliances took place at the end of the First Congo War, but that alliances remain broadly stable thereafter (cf. Prunier 2011). However, one might worry that this broad pattern may not apply to small groups, whose behavior is not equally well-documented by the conflict literature. To get a sense of the potential importance of the problem, we search for instances in which there is a clear switch in the nature of a dyadic relationship in ACLED. A clear evidence of structural change would be that there exists a threshold year $T \in (1998 - 2010)$ such that two groups would be classified as allies (enemies) if one restricted attention to the years up to T, while the same two groups would be classified as enemies (allies) if one considered the years after T. We have eight dyads, a mere 0.2% of the total 3570 dyads that conform with this pattern. This is a strong indication that in most of the 1% dyads for which an ambiguity arises, this ambiguity is due to occasional clashes between troops (or one-shot tactical alliances within a single battle) rather than to structural changes in system of alliances and enmities. This is reassuring for our assumption that the network is time-invariant.[26]

The second concern is that the construction of the network partly relies on the same ACLED data that we use to measure the outcome variable (i.e fighting). Here, let us emphasize two important differences. First, for the network we exploit the bilateral (dyadic) information which is not used to construct the outcome variable. Second, the network is time-invariant, whereas our econometric analysis exploits the time variations in fighting efforts, controlling for group fixed effects, as discussed in more detail below. To further alleviate this concern, in Section 4 we build

[25] Given that in our theoretical setting all groups are competing, by definition, for the same prize, we require at least two instances of fighting against each other to code two groups as *enemies*. As shown below, our results are robust to alternative coding where groups are *enemies* when they fight against each other in at least one instance.

[26] To further alleviate this concern, in the robustness section we restrict the sample to 1998-2007, since there is anedoctal evidence that after 2007 some militias were wiped out or absorbed by the DRC Army. The results are robust.

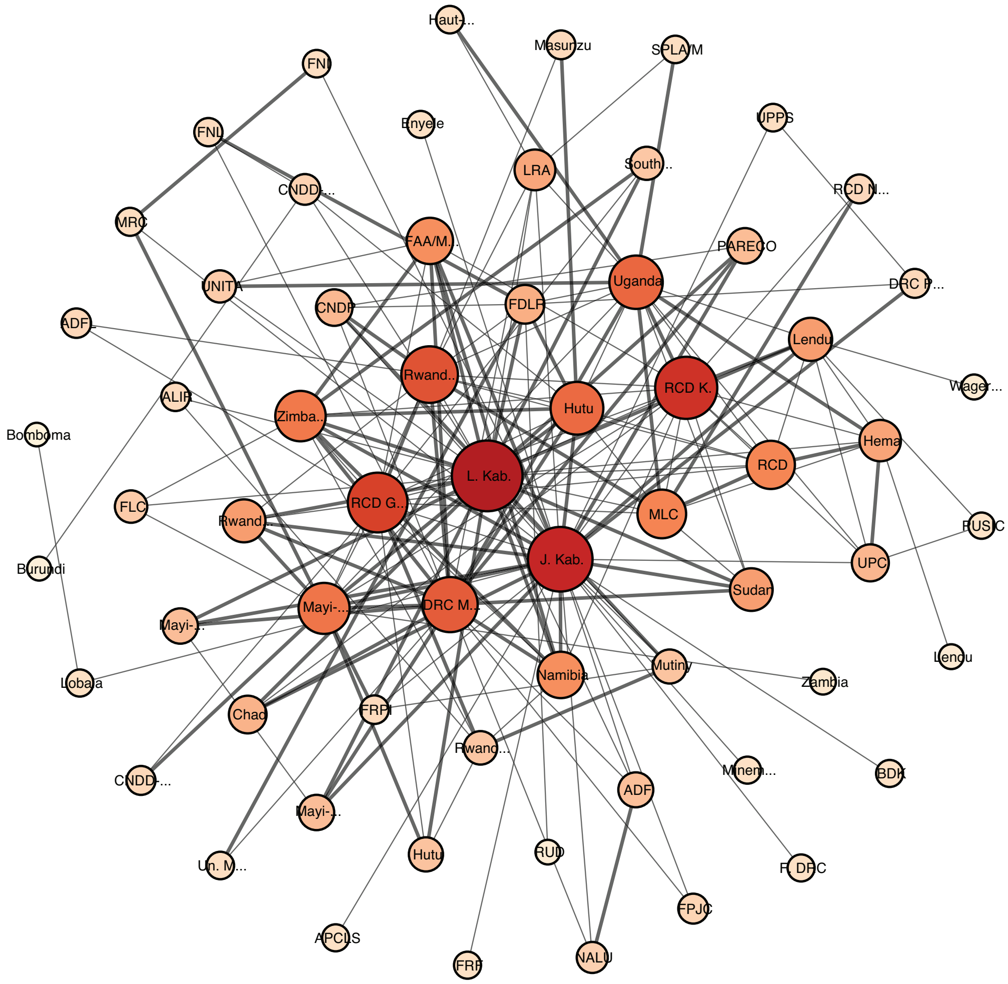


Figure 6: The figure shows the network of alliances and conflicts in the DRC. Thick lines indicate alliances, while thin lines indicate enmities. The nodes' colors and sizes represent their centrality (cf. Section 2.2).

the network using SIPRI and ACLED data from the period 1998-2001, while the outcome variable uses the information for the period 2002-2010. This comes at a high cost in term of information loss since the conflict attains its highest intensity in the initial years. The results are robust, albeit less precisely estimated.

Third, we are likely to miss some network links. It is likely that we code as neutral some dyads that are in fact allies or enemies but did not participate into common fighting events (e.g. due to spatial distance). Such missing links create measurement errors that can bias the estimates of the fighting externalities (Chandrasekhar and Lewis, 2011). The strategy to tackle this issue is twofold. On the one hand, we check the robustness of our results to alternative coding rules of alliances and enmities.[27] On the other hand, we perform Monte Carlo simulation to assess the bias associated with an imperfect observation of missing links.

Figure 6 illustrates the network of alliances and rivalries in DRC. Not surprisingly, the armed forces of DRC have the highest centrality. The other groups with a high centrality are the Rally for Congolese Democracy (Goma and Kisangani), the Mayi-Mayi militias, and the foreign armies of Rwanda, Uganda, Zimbabwe, Namibia, Sudan, and Angola. All these groups are known to have played a salient role in the Second Congo War.

[27] We consider a variety of alternative coding rules of alliances and enmities. In addition, we check the robustness of the results to using alternative data sources. In particular, we supplement our information with that provided in the "Non-State Actor Data" of Cunningham, Gleditsch and Salehyan (2013).

3.4 Econometric Model

The basis of our empirical analysis is Section 2.6, allowing for exogenous sources of heterogeneity in the OP of groups. Equation (13) can be turned into an econometric equation by assuming that the individual shocks $\tilde{\varphi}_i$ comprise both observable and unobservable shifters. More formally, we assume that $\tilde{\varphi}_i = \mathbf{z}_i' \boldsymbol{\alpha} + \epsilon_i$, where $\mathbf{z}_i$ is a vector of group-specific observable characteristics, and ϵ_i is an unobserved shifter. Replacing x_i and φ_i by their respective equilibrium values, yields the following structural equation:

$$x_i^* = \varphi_i^* (G) - \beta \sum_{j=1}^{n} a_{ij}^{+} x_j^* + \gamma \sum_{j=1}^{n} a_{ij}^{-} x_j^* - \mathbf{z}_i' \boldsymbol{\alpha} - \epsilon_i. \tag{15}$$

It is important to recall that φ_i^* is fully characterized by the externality parameters and the time-invariant network structure, $(\beta, \gamma, d_i^-, d_i^+)$, and is independent of the realizations of individual shocks $(\mathbf{z}_i, \epsilon_i)$ (see equation 4 and Section 2.6). Our goal is to estimate the network parameters β and γ. The estimation is subject to a simultaneity or reflection problem (Manski 1993; Boucher *et al.* 2012), a common challenge in the estimation of network externalities. A related issue in this class of models is that it is difficult to separate contextual effects, i.e., the influence of players' characteristics, from endogenous effects, i.e., the effect of outcome variables via network externalities. In our model, the endogenous effect is associated with the fighting effort exerted by a group's allies and enemies. Although our theory postulates no contextual effect, it is plausible that omitted variables affecting x_i^* are spatially correlated, implying that one cannot safely assume spatial independence of ϵ_i. Ignoring this problem might yield inconsistent estimates of the spillover parameters.

The reflection problem can be tackled by using instruments to obtain consistent estimates of the spillover effects. For instance, in a recent study on public good provision in a network of Colombian municipalities, Acemoglu, Garcia-Jimeno and Robinson (2014) use as instruments historical characteristics of local municipalities which are argued to be spatially uncorrelated. In our case, it is difficult to single out time-invariant group characteristics that affect the fighting efforts of a group's allies or enemies without invalidating the exclusion restriction. For instance, cultural or ethnic characteristics of group i are likely to be shared by its allies. For this reason, we take the alternative route of identifying the model out of exogenous time-varying shifters affecting the fighting intensity of allies and enemies over time. This panel approach has the advantage that we can difference out any time-invariant heterogeneity, thereby eliminating the problem of correlated fixed effects.

Panel Specification – We maintain the assumption of an exogenous time-invariant network, and assume that the conflict repeats itself over several years. We abstract from reputational effects, and regard each period as a one-shot game. These are strong assumptions, but they are necessary to retain tractability. The variation over time in conflict intensity is driven by the realization of group- and time-specific shocks, amplified or offset by the endogenous response of the players which, in turn, hinges on the network structure. More formally, we allow both x_i^* and $\tilde{\varphi}_i$ to be time-varying:

$$\tilde{\varphi}_{it} = \mathbf{z}_{it}' \boldsymbol{\alpha} + e_i + \epsilon_{it}. \tag{16}$$

Here, $\mathbf{z}_{it}$ is a vector of observable shocks with coefficients $\boldsymbol{\alpha}$, e_i is a vector of (unobservable) time-invariant group-specific characteristics, and ϵ_{it} is a i.i.d., zero-mean unobservable shock. Rainfall measures are examples of observable shifters $\mathbf{z}_{it}$ that will be key for identification. The panel analogue of equation (15) can then be written as:

$$\text{FIGHT}_{it} = \text{FE}_i - \beta \times \text{FIGHT}_{it}^{\text{ALL}} + \gamma \times \text{FIGHT}_{it}^{\text{ENE}} - \mathbf{z}_{it}' \boldsymbol{\alpha} - \epsilon_{it}. \tag{17}$$

where

$$\begin{aligned}
\textsc{Fight}_{it} &= x^*_{it} \\
\textsc{Fight}^{\text{ALL}}_{it} &= \sum_{j=1}^{n} a^+_{ij} x^*_{jt}, \\
\textsc{Fight}^{\text{ENE}}_{it} &= \sum_{j=1}^{n} a^-_{ij} x^*_{jt}, \\
\textsc{fe}_i &= -\varphi^*_i(G) - e_i. \qquad (18)
\end{aligned}$$

The panel dimension allows us to filter out any time-invariant correlated effects by including group fixed effects $\textsc{fe}_i$. However, due to the reflection problem discussed above, the two covariates $\textsc{Fight}^{\text{ALL}}_{it}$ (*Total Fighting of Allies*) and $\textsc{Fight}^{\text{ENE}}_{it}$ (*Total Fighting of Enemies*) are correlated with the error terms. In other words, there is an endogeneity problem because the effort of group i's allies and enemies are affected by group i's effort. Thus, OLS estimates are inconsistent.

The problem can be addressed by an instrumental variable strategy. This requires identifying exogenous sources of variations in the fighting efforts of group i's allies and enemies that do not influence group i's fighting effort directly. To this aim, we use time-varying climatic shocks (rainfall) impacting the homelands of armed groups. In line with the empirical literature and historical case studies (Dell 2012), we focus on local rainfall as a time-varying shifter of OP, and hence the fighting effort of allies and enemies. We expect groups affected by positive rainfall shocks to fight less, since local rainfall increases the agricultural surplus, thereby pushing up the reservation wages of productive labor and, hence, the opportunity cost of fighting. This channel linking rainfall to conflict has been documented, among others, by Jia (2014), Hidalgo *et al.* (2010), Miguel *et al.* (2004), Vanden Eynde (2011).

To be a valid instrument, rainfall in the homelands of the allies (enemies) must be correlated with the allies' (enemies') fighting efforts. We show below that this is so in the data. The exclusion restriction requires that rainfall in the homelands of group i's allies and enemies have no direct effect on fighting efforts of group i. Although rainfall is likely to be spatially correlated, due to the proximity of the homelands of allied or enemy groups, this is not a problem since we control for the rainfall in group i's homeland in the second-stage regression. For instance, suppose that group i has a single enemy, group k, and that the two groups live in adjacent homelands. Rainfall in k's homeland is correlated with rainfall in i's homeland. However, rainfall in k's homeland is a valid instrument for k's fighting effort, as long as rainfall in i's homeland is included as a non-excluded instrument. A potential issue arises if rainfall is measured with error, and measurement error has a non-classical nature. We tackle this issue below in the robustness analysis.

Potential violation of the exclusion restriction could arise if there were intense within-country trade. For instance, a drought destroying crops in Western Congo could translate into strong price increases of agricultural products throughout the entire DRC, thereby affecting fighting in the Eastern part of the country. Such a channel may be important in a well-integrated country with large domestic trade. Yet, in a very poor country like the DRC with a disintegrating government, very lacunary transport infrastructure and a disastrous security situation, effective transport costs are high and inter-regional trade is limited for most goods. The result is a very localized economy dominated by subsistence farming. Hence, spillovers through trade are unlikely to be large.

3.5 Estimates of the Fighting Externalities

We now present the estimates of the system of structural equations (17) based on our panel of 85 armed groups over 1998-2010. In all specifications, we include group fixed effects and year dummies, and cluster standard errors at the group level.

Table 2 displays the baseline results. Column (1) starts with an OLS specification. We control for current and lagged rainfall at the centroid of group's homeland in a flexible way, allowing for both linear and quadratic terms. *Total Fighting of Enemies* increases a group's fighting effort, whereas *Total Fighting of Allies* decreases it. These results conform with the prediction of the theory, although the coefficient of *Total Fighting of Allies* is not statistically significant.

In column (2) we add a set of time-varying control variables by assuming that common (unobserved) shocks have heterogeneous effects across groups that differ by some observable characteristics. More specifically, we build three time-invariant group characteristics and interact each of them with year dummies. The first characteristic corresponds to a binary variable capturing whether a group is a government organization (as opposed to a non-state actor). State organizations benefit from stable funding and legitimacy conveyed by the law, and thus tend to be larger and better equipped than local militias and rebel groups. For these reasons, government armies are likely to be affected differently from informal armed groups by global economic and political shocks. The two other characteristics are proxies for groups' size.[28] Both are binary variables coding, respectively, for groups with at least 10 enemies, and for groups involved in at least 20 violent events per year *outside* of the DRC (each variable captures groups between the top 5-10 percent of the sample). The results are not sensitive to using different thresholds or to dropping either of them. The estimates of column (2) are very similar to those of column (1).

Due to the reflection problem discussed above, OLS estimates are inconsistent. Therefore, in the next columns, we replicate specifications of columns (1)-(2) in a 2SLS setup using first a restricted set of instruments (columns 3 and 4), and then an enlarged one (columns 5 and 6). The related first stage regressions are reported in Table 3, where, for presentational purposes, only the coefficients of the excluded instruments are displayed. Remarkably, there is a stable pattern across all first-stage regressions by which rainfall in the enemies' homelands affects negatively the fighting effort of the enemy groups (and not that of allied groups), whereas rainfall in the allies' homelands decreases the fighting effort of the allied group (and not that of the enemies). This pattern conforms with the theoretical predictions.

In Columns (3) and (4) of Table 2, *Total Fighting of Enemies* and *Total Fighting of Allies* are instrumented, respectively, by the one-year lag of average rainfall in the homelands of enemies and allies, including both a linear and a quadratic term in the first-stage regressions. In column (3), the coefficients of the variables of interest have the expected sign and are significant at the 5% level. The coefficients are larger than in the OLS regressions, suggesting that the non-instrumented specification suffers from an under-estimation bias due to the omission of the network externalities. Column (4) includes the large set of time-varying controls. This leads to a weak instrument problem and the coefficient of *Total Fighting of Enemies* turns insignificant.

Columns (1)-(4) of Table 3 display the first stage regressions for *Total Fighting of Enemies* and *Total Fighting of Allies* corresponding to columns (3)-(4) in Table 2. We see that the instruments have statistically significant coefficients, with the expected sign, but their overall statistical power is borderline in absence of time-varying controls (with a Kleibergen-Paap F-stat equal to 9.34) and becomes weak once controls are included (F-stat equal to 6.65). This issue is solved below by expanding the set of instruments. The null hypothesis of the Hansen J test is not rejected

[28] We have no good data for troop size. Such data exist only for a small subset of the groups, see the International Institute of Strategic Studies (IISS) or the Small Arms Survey (SAS).

Table 2: Baseline regressions (second stage).

	Dependent variable: Total Fighting					
	(1)	(2)	(3)	(4)	(5)	(6)
Tot. Fight. Enemies	0.09***	0.06***	0.12**	0.08	0.14***	0.09**
	(0.02)	(0.02)	(0.05)	(0.07)	(0.05)	(0.05)
Tot. Fight Allies	-0.01	-0.02	-0.16**	-0.17**	-0.15*	-0.14**
	(0.02)	(0.01)	(0.07)	(0.08)	(0.08)	(0.07)
Group FE, annual TE, rain controls	Yes	Yes	Yes	Yes	Yes	Yes
Additional controls	No	Yes	No	Yes	No	Yes
Estimator	OLS	OLS	IV	IV	IV	IV
Set of Instrument Variables	n.a.	n.a.	Restricted	Restricted	Full	Full
Observations	1190	1190	1190	1190	1105	1105
R-squared	0.340	0.415	0.241	0.323	0.254	0.364

Notes: An observation is a given armed group in a given year. The panel contains 85 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

(with p-value of 0.53) indicating that the overidentification restrictions are valid. Interestingly, the fighting effort of enemies is highly correlated with rainfalls in enemies homeland while the total fighting effort of the allies is highly correlated with the rainfall in allies homeland. This reassuring pattern is robust across all columns of Table 3. Moreover, consistent with the findings of literature discussed above, a higher rainfall in period $t-1$ predicts a lower fighting effort. This pattern is confirmed in specifications including both the current and lagged rainfall.

In column (5) of Table 2, we consider a larger set of instruments comprising current-year rainfall (linear and quadratic terms) as well as current and lagged rainfalls of degree 2 neighbors (i.e. enemies of enemies and of enemies of allies).[29] The second-stage coefficients are stable and significant. The statistical power of the first stage (columns (5)-(6) in Table 3) is now high above the conventional threshold of 10 (F-stat at 19.16).

Column (6) of Table 2 – our preferred specification – includes the additional time-varying controls and uses the expanded set of instruments. The two coefficients of interest have the expected signs and are statistically significant at the 5% level. The statistical power of the instruments in the first stage (columns (7)-(8) in Table 3) is large, with a Kleibergen-Paap F-statistic equal to 13.71. The estimates of the fighting externalities are quantitatively large. A one standard deviation (s.d.) increase in *Total Fighting of Enemies* (93 violent events) translates into a 0.37 s.d. increase in the Fighting Effort of the group (+8.4 violent events). A one s.d. increase in *Total Fighting of Allies* (75 violent events) translates into a 0.46 s.d. decrease in the Fighting Effort of the group (-10.5 violent events). We also check that, conditional on estimates of β and γ, the second-order condition, (3), holds true for *all* groups in conflict. This guarantees that we estimate an interior equilibrium where the invertibility condition of Proposition 1 is also satisfied.

[29] When we use the current and past average rainfall in enemies' and allies' homelands as instruments, we also control for the current and past average rainfall in the own group homeland in the second-stage regression. This is important, since the rainfall in enemies' and allies' homelands is correlated with the rainfall in the own group homeland. Omitting the latter would lead to a violation of the exclusion restriction.

Table 3: Baseline regressions (first stage).

	IV regression of column (3)		IV regression of column (4)		IV regression of column (5)		IV regression of column (6)	
Dep. Variable:	Tot.Fi. Enemy	Tot.Fi.Allied	Tot.Fi. Enemy	Tot.Fi.Allied	Tot.Fi. Enemy	Tot.Fi.Allied	Tot.Fi. Enemy	Tot.Fi.Allied
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Rain (t-1) Enemies	**-1.45*** **	0.03	**-1.30*** **	0.07	**-1.34*** **	0.18	**-1.20*** **	0.15
	(0.23)	(0.10)	**(0.27)**	(0.09)	**(0.24)**	(0.11)	**(0.30)**	(0.12)
Sq. Rain (t-1) Ene.	0.00***	0.00	0.00***	0.00	0.00***	0.00	0.00***	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Rain (t-1) Allies	-0.01	**-1.02*** **	-0.03	**-0.99*** **	-0.26	**-0.74*** **	-0.32	**-0.73*** **
	(0.22)	**(0.15)**	(0.18)	**(0.14)**	(0.29)	**(0.19)**	(0.29)	**(0.19)**
Sq. Rain (t-1) Alli.	0.00	0.00***	0.00	0.00***	0.00	0.00***	0.00	0.00***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Current Rain Enemies					**-0.97*** **	0.10	**-0.99*** **	0.08
					(0.25)	(0.13)	**(0.27)**	(0.13)
Sq. Curr. Rain Ene.					0.00***	-.00	0.00	0.00
					(0.00)	(0.00)	(0.00)	(0.00)
Current Rain Allies					-0.21	**-0.67*** **	-0.30	**-0.66*** **
					(0.18)	**(0.12)**	(0.20)	**(0.12)**
Sq. Curr. Rain Alli.					0.00	0.00***	0.00*	0.00***
					(0.00)	(0.00)	(0.00)	(0.00)
Current rain enemies of enemies					-0.11***	-0.05*	-0.10**	-0.04
					(0.04)	(0.03)	(0.04)	(0.03)
Sq. current rain enemies of enemies					0.00	0.00	0.00	0.00
					(0.00)	(0.00)	(0.00)	(0.00)
Current rain enemies of allies					-0.12	-0.03	-0.08	-0.04
					(0.08)	(0.04)	(0.08)	(0.04)
Sq. current rain enemies of allies					0.00*	0.00	0.00	0.00
					(0.00)	(0.00)	(0.00)	(0.00)
Rain enemies of enemies (t-1)					-0.15***	-0.06**	-0.19***	-0.07**
					(0.05)	(0.03)	(0.05)	(0.03)
Sq. rain enemies of enemies (t-1)					0.00*	0.00	0.00**	0.00**
					(0.00)	(0.00)	(0.00)	(0.00)
Rain enemies of allies (t-1)					0.02	-0.07	0.04	-0.04
					(0.10)	(0.05)	(0.11)	(0.04)
Sq. rain enemies of allies (t-1)					0.00	0.00	0.00	0.00
					(0.00)	(0.00)	(0.00)	(0.00)
F-Stat (Kleibergen-Papp)	9.34	9.34	6.65	6.65	19.16	19.16	13.71	13.71
Hansen J (p-value)	0.53	0.53	0.19	0.19	0.19	0.19	0.26	0.26
Observations	1190	1190	1190	1190	1105	1105	1105	1105
R-squared	0.47	0.63	0.57	0.65	0.56	0.67	0.63	0.69

Notes: An observation is a given armed group in a given year. The panel contains 85 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 4: Sample splitting.

	Dependent variable: Total Fighting					
	(1)	(2)	(3)	(4)	(5)	(6)
Tot. Fight. Enemies	0.12***	0.10**	0.17***	0.03	0.04	0.09*
	(0.04)	(0.04)	(0.06)	(0.13)	(0.04)	(0.05)
Tot. Fight Allies	0.02	0.00	-0.19**	-0.18***	-0.02	-0.02
	(0.03)	(0.03)	(0.08)	(0.06)	(0.04)	(0.04)
Group FE, annual TE, rain controls	Yes	Yes	Yes	Yes	Yes	Yes
Additional controls	No	Yes	No	Yes	No	Yes
Estimator	OLS	OLS	IV	IV	IV	IV
Set of Instrument Variables	n.a.	n.a.	Restricted	Restricted	Full	Full
F-Stat (Kleibergen-Papp)	n.a.	n.a.	6.03	4.09	100.00	26.02
Hansen J (p-value)	n.a.	n.a.	0.38	0.08	0.55	0.09
Observations	850	850	850	850	765	765
R-squared	0.42	0.48	0.28	0.38	0.43	0.51

Notes: An observation is a given armed group in a given year. The panel contains 85 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

4 Robustness Analysis

In this section, we perform a variety of robustness checks.

4.1 Alternative Specifications

In our benchmark regressions the same data (i.e., the fighting events from ACLED) are used to measure the network structure and the fighting effort. This is in principle not an issue, since the identification exploits the time variation in the number of fighting events, whereas the network structure is time-invariant. However, it is possible to separate the two variables more sharply by estimating the network from the earlier part of the sample, and then studying the dynamics of the conflict using the later years of the conflict.[30] More precisely, we use the information for the subperiod 1998-2001 to estimate the network links, and the panel 2002-2010 to estimate the spillover coefficients. The results are reported in Table 4, the analogue of Table 2. The results are similar, albeit more fragile due to the one-third reduction in sample size. In particular, the specification of column (3) yields similar results. However, in our preferred specification of column (6), which is more demanding, the coefficient of *Total Fighting of Allies* turns insignificant. It is reassuring that the signs of the 2SLS second stage coefficients are consistent in all cases with the theoretical predictions.

The fact that the results are somewhat weaker is no surprise. On the one hand, the subperiod 1998-2001 is very intense in violent events – roughly a third of the total ACLED events in our sample occurs in this subperiod. Therefore, identifying the model out of the variation over time during 2002-2010 is bound to yield less precise estimates. On the other hand, the network of enmities and alliances also is measured with error, and many links are likely to be missed (we show below that indeed measurement error in the network links yields an attenuation bias in our model). This problem becomes more severe when we use only four years of data to estimate the links.

[30] It is impossible to estimate the network using data prior to 1998, since there was a major reshuffling of tactical alliances between the First and the Second Congo War, as discussed in section 3.1.

In all following robustness checks (Tables 5-6), we report the results of second-stage regressions corresponding to our preferred (*benchmark*) specification of column (6) in Table 2.

Table 5 considers alternative instrumentation and network construction strategies. In column (1), we use only the rainfalls in the homeland of degree 2 neighbors (i.e., the rain of enemies of enemies and of enemies of allies) as excluded instruments, following Bramoullé, Djebbari and Fortin (2009).[31] The point estimates in the second-stage regression are similar to our benchmark results. However, they are estimated less precisely, and the coefficients of *Total Fighting of Allies* turns marginally insignificant. Moreover, we face a serious weak instrument problem in the first-stage regression (where the F-stat is 3.8). This is not surprising since the degree one rainfalls are controlled for both in the first- and in the second-stage regressions.

In column (2) we follow a conservative identification strategy using the degree one and two rainfalls in the groups' historical ethnic homelands as excluded instruments. By construction these homelands do not overlap spatially, which guarantees that the rainfalls associated with different groups do not overlap either. As a first step we link as many armed groups as possible to a corresponding underlying main ethnic group. This is typically the ethnic affiliation of most fighters, or at least of their leadership circle. For example, the Lord's Resistance Army is linked to the Acholi ethnic group. We find an unambiguous match for 94% of all armed groups, and drop the remaining 6%. As a second step, we compute the rainfall averages on the polygons of all ethnic groups, using the digitalized version by Nunn and Wantchekon (2011) of the map of historical ethnic group homelands from Murdock (1959). Given that rainfall in ethnic homelands is an imperfect proxy for rainfall observed by groups in their actual current territory, the power of the first stage falls, and we suffer from a weak instrument problem. Yet, the point estimates of the second-stage regression are remarkably similar to the benchmark 2SLS results; only the statistical significance of *Total Fighting of Allies* drops.

In column (3)-(4) we address the concern that the results may be driven by many small groups that have a limited role in the overall conflict. In column (3) we include only groups that have at least one enemy. This results in a sizeable reduction in the sample size. The coefficients are on average slightly larger in absolute value than in the baseline regression, and are both significant at the 5% level. The F-stats of the first stage are about the threshold of 10. In column (4) we restrict the regressions to groups that have at least one enemy and at least one ally, in order to focus on the most salient armed forces. Reassuringly, the results are quantitatively larger (again, in absolute value) than in the benchmark specification, and both coefficients continue to be highly significant. In summary, these robustness checks show that restricting attention to large players yields larger and more precisely estimated coefficients.

In column (5), we exclude all events involving group i when computing the total fighting efforts of allies and enemies of group i. If, for example, the enemies of the LRA are involved in 100 fighting events in year 2000, out of which 30 involve the LRA, then the measure of *Total Fighting of Enemies* used in the regression would take the value of 70. The results are similar, although the coefficient of *Total Fighting of Enemies* turns marginally insignificant.

In column (6), we also control for the total fighting of neutral groups, i.e., those classified neither as allies nor as enemies. Our theory predicts that the coefficient of neutrals should be zero. In line with the theory, the coefficients of *Total Fighting of Enemies* and *Total Fighting of Allies* are virtually unchanged, and the coefficient of *Total Fighting of Neutrals* is zero, as predicted by the theory.

[31] Note that, contrary to their model, in our theory there is no reason why an instrumentation based on first-order links should yield inconsistent estimates. As discussed above, the case for our regressions to be contaminated by contextual effects is weak in our panel regression, since time-invariant contextual effects are differenced out.

Finally, in column (7) we control for the lagged total fighting effort of both enemies and allied. The coefficient on the lagged variables are insignificant, whereas the coefficients of interest are hardly changed.

In Table 6, we perform a battery of additional robustness checks where we either restrict the set of fighting events considered (columns (1)-(3)), or we use alternative coding of enmities and alliances in the construction of the network (columns (4)-(7)).

The motivation for the robustness checks in columns (1)-(2) is that ideally fighting intensity should weight the importance of the events in which each group is involved. However, detailed information about the size of each event (e.g., the number of casualties) is far too sparse for us to be able to run weighted regressions. Nevertheless, we have access to some binary coding from ACLED. In column (1) we drop all events with a number fatalities equal to zero (i.e. we retain the events associated with a positive number of fatalities or for which the information about the number of fatalities is missing). The coefficients of interest remain very similar to the ones of the baseline regression of column (6) of Table 2, and both variables are significant at the 5% level. In column (2) we use only events that are classified in ACLED as battles, ignoring other events. The estimated coefficients have the same sign as in the benchmark case, and are quantitatively similar. Interestingly, the estimated externalities are stronger in column (2), indicating that restricting attention to battles, which are more important events than small riots, increases the point estimates.

In column (3) we restrict the sample to 1998-2007. The motivation is that after 2007 a few militias were either annihilated (e.g., the Allied Democratic Front) or absorbed by larger combatant groups (most notably, the DRC Army). In spite of the limited number of such episodes, the disappearances or mergers of some groups contradicts the assumption that the network is time-invariant. Reassuringly, the estimates are of similar magnitude than in the baseline regression of column (6) of Table 2 and are statistically significant.

In column (4) we code as enemies (instead of neutrals) groups that normally fight against each other, but that were observed occasionally fighting on opposite sides. The magnitude of the coefficients remains similar, although *Total Fighting of Enemies* now falls short of the 10% significance threshold. In column (5) we code a dyad of armed groups as enemies if they fight each other at least once and never fight together on the same side (recall that in the benchmark we required two groups to fight each other twice or more). The coefficients of the two variables remain of similar magnitude and are highly significant. In contrast, in column (6) we follow a more restrictive rule: we code two groups as enemies if they are observed fighting on opposite sides in at least two events (but never on the same side), and code two groups as allies if they have been observed fighting together on the same side at least twice (but never against each other). The two variables of interest continue to have the expected sign and are statistically significantly different from zero. Finally, in column (7) we add also links among groups listed in Cunningham *et al.* (2013). Our results are robust with both coefficients of interest being significant and of the expected sign.

In Table 7 we perform a series of robustness checks with respect to the definition of groups. One might worry that the assumption that groups act non-cooperatively is inappropriate for a number of strong alliances that should be better treated as unitary coalitions. In our benchmark analysis we have followed the rule of treating groups as separate entities whenever they are classified as such by ACLED. This agnostic way of proceeding has the advantage of not requiring any discretional coding decision.[32] However, it is useful to check the robustness of our results in this dimension. Thus, in column (1) of Table 7 we treat all different fractions of the Rally of Congolese Democracy

[32]Further, note that even among strongly allied groups there is sometimes in-fighting. For example, the clashes between RCD's Goma and RCD's Kisangani fraction are infamous, as are the fights between different Mayi-May militias.

Table 5: Robustness to alternative IVs and network construction.

	Dependent variable: Total Fighting						
	(1) Degree 2 only	(2) Ethnic IV	(3) With at least 1 enemy	(4) With at least 1 ally & 1 en.	(5) Excluding bilat. evts.	(6) With neutrals	(7) With lags
Tot. Fight. Enemies	0.12**	0.08**	0.09**	0.11**	0.05	0.11**	0.08**
	(0.06)	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.04)
Tot. Fight Allies	-0.26	-0.15	-0.18**	-0.26**	-0.14**	-0.16*	-0.14**
	(0.18)	(0.13)	(0.09)	(0.13)	(0.06)	(0.08)	(0.07)
Tot. Fight Neutrals						0.00	
						(0.01)	
Tot. Fight. Enemies (t-1)							0.00
							(0.02)
Tot. Fight Allies (t-1)							0.03
							(0.02)
Group FE, annual TE, add. cont.	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Zones used for IV construction	Group pr.	Eth. home	Group pr.	Group pr.	Group pr.	Group pr.	Group pr.
Excluded instruments used	Deg 2	Deg 1&2	Deg 1&2	Deg 1&2	Deg 1&2	Deg 1&2	Deg 1&2
	Lag 0&1	Lag 0&1	Lag 0&1	Lag 0&1	Lag 0&1	Lag 0&1	Lag 0&1
F-Stat (Kleibergen-Papp)	3.82	3.55	9.95	7.87	12.03	15.72	24.73
Hansen J (p-value)	0.74	0.16	0.34	0.62	0.23	0.29	0.54
Observations	819	1027	819	559	1105	1105	1020
R-squared	0.28	0.36	0.34	0.27	0.35	0.33	0.39

Notes: An observation is a given armed group in a given year. The panel contains 85 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 6: Robustness to alternative definitions of events and links.

	Dependent variable: Total Fighting						
	(1) Only violent events	(2) Only battles	(3) Only until year 2007	(4) Code incons. as enemy	(5) Code as enemy when opp. in at least 1 event	(6) Code as ally only when at least 2 joint events	(7) Add links from Cunningham et al.
Tot. Fight. Enemies	0.09**	0.11**	0.09**	0.06	0.11***	0.11**	0.10*
	(0.05)	(0.05)	(0.04)	(0.05)	(0.03)	(0.04)	(0.06)
Tot. Fight Allies	-0.14**	-0.17**	-0.13*	-0.10**	-0.19**	-0.10*	-0.12*
	(0.07)	(0.08)	(0.08)	(0.05)	(0.09)	(0.05)	(0.07)
Group FE, annual TE, add cont.	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-Stat (Kleibergen-Papp)	13.71	12.61	9.10	10.76	7.58	19.05	16.04
Hansen J (p-value)	0.26	0.45	0.77	0.51	0.89	0.34	0.30
Observations	1105	1066	850	1105	1105	1105	1105
R-squared	0.36	0.34	0.40	0.43	0.27	0.40	0.38

Notes: An observation is a given armed group in a given year. The panel contains 85 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * p < 0.1, ** p < 0.05, *** p < 0.01.

(RCD) as one single actor. Our variables of interest still have the expected sign and are significant at the 5% level. In column (2) we merge the RCD's Goma fraction (RCD-G) with its main ally, Rwanda, and similarly we merge RCD's Kisangani fraction (RCD-K) with its main ally, Uganda. In column (3) we merge the various DRC army fractions into two main actors. Further, in column (4) we merge all local Mayi-Mayi militias into one single actor. In column (5) all Rwandan military fractions are treated as a single group. Finally, in column (6) we treat as separate actors all mutinies of the DRC army. The results are robust across all specifications.

4.2 Measurement Error in Rainfall

A concern with our IV strategy is that the rainfall variable may be subject to non-classical measurement error. In particular, fighting activities may destroy rain gauges located in battlefields. As a result, our gauge-based GPCC measure might systematically underreport precipitations in war zones. The issue is twofold: first, mis-measurement may result in a spurious negative correlation between rainfall and fighting in the first-stage regression.[33] Second, our identification hinges on rainfall in the homelands of group i's enemies/allies having no direct effect on group i's fighting effort after conditioning on the rainfall in group i's homelands. However, the exclusion restriction would be invalidated if the measurement error in the instruments were correlated with group i's fighting effort.

To study this potential problem, we consider satellite rainfall estimates from TRMM or GPCP (see the data description in Section 3.2). Clearly satellite-based measurements are not affected by the dynamics of conflict. However, they provide less direct and far less accurate rainfall estimates than do gauges.[34] Therefore, it is not surprising that, if we use satellite rainfall data instead of gauge-based data as instruments, we run into a severe weak instrument problem. However, the satellite estimates can be used to infer whether gauged-based measures are biased. To this aim, consider the following simple model:

$$\begin{aligned} \text{RAIN}_{ct}^{\text{SAT}} &= \psi_c^{\text{SAT}} + \text{RAIN}_{ct} + v_{ct}^{\text{SAT}} && (19) \\ \text{RAIN}_{ct}^{\text{GAU}} &= \psi_c^{\text{GAU}} + \text{RAIN}_{ct} + \tilde{v}_{ct}^{\text{GAU}} && (20) \end{aligned}$$

where c denotes the grid cell at which rainfall is measured, RAIN_{ct} is the true (unobservable) rainfall, and v_{ct}^{SAT} and $\tilde{v}_{ct}^{\text{GAU}}$ are the measurement errors. v_{ct}^{SAT} is assumed to be i.i.d.. The error term of the gauge measure is potentially subject to violence-driven measurement error. This possibility is allowed by letting $\tilde{v}_{ct}^{\text{GAU}} = \xi \times \text{VIOLENCE}_{ct} + v_{ct}^{\text{GAU}}$ where v_{ct}^{GAU} is an i.i.d error term. One can eliminate RAIN_{ct} from the previous system of equations and obtain:

$$\text{RAIN}_{ct}^{\text{GAU}} = \psi_c + \text{RAIN}_{ct}^{\text{SAT}} + \xi \times \text{VIOLENCE}_{ct} + \nu_{ct} \qquad (21)$$

where $\psi_c = \psi_c^{\text{GAU}} - \psi_c^{\text{SAT}}$ and $\nu_{ct} = v_{ct}^{\text{GAU}} - v_{ct}^{\text{SAT}}$ are, respectively, a grid-cell fixed effect and an i.i.d. disturbance. Our null hypothesis is that $\xi = 0$. If $\xi \neq 0$, the gauge-based measure suffers with non-classical measurement error.

We run a regression based on equation (21), measuring violence by the number of conflicts in ACLED. Table 8 summarizes the results. Columns (1)-(4) report the results when satellite-based rainfall measures are retrieved from TRMM. Column (1) is a cross-sectional specification; Column

[33] Remember, though, that we also use lagged rain to predict current fighting intensity.

[34] Romilly and Gebremichael (2011) discuss the shortcomings of satellite-based rainfall estimates. On the one hand, satellite rainfall estimates are contaminated by sources such as temporal sampling, instrument and algorithm error. On the other hand, a number of studies based on U.S. data document that their performance varies systematically with season, region and elevation, resulting in potentially severe biases.

Table 7: Robustness to alternative group definitions.

	Dependent variable: Total Fighting					
	(1) Merging all RCD fractions into a single actor	(2) Merging RCD-G to Rwanda and RCD-K to Uganda	(3) Merging all DRC government groups into 2 main fractions	(4) Merging all Mayi-Mayi local militias into a single actor	(5) Merge Rwandan military fractions into a single actor	(6) Treating mutiny events as separate actors
Tot. Fight. Enemies	0.12**	0.08*	0.07*	0.10**	0.09**	0.10**
	(0.05)	(0.05)	(0.04)	(0.05)	(0.04)	(0.05)
Tot. Fight Allies	-0.18**	-0.10*	-0.12**	-0.18*	-0.12**	-0.14*
	(0.08)	(0.06)	(0.06)	(0.10)	(0.06)	(0.07)
Group FE, annual TE, add cont.	Yes	Yes	Yes	Yes	Yes	Yes
F-Stat (Kleibergen-Papp)	13.86	16.21	11.07	8.44	15.58	14.21
Hansen J (p-value)	0.25	0.50	0.37	0.64	0.27	0.54
Observations	1053	1066	1079	1014	1092	1144
R-squared	0.38	0.49	0.37	0.35	0.42	0.36

Notes: An observation is a given armed group in a given year. The panel contains 85 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

(2) includes grid-cell fixed effects – consistent with equation (21). In Columns (3) and (4) we consider a log-linear specification where the two rainfall measures are log-scaled; this corresponds to a multiplicatively separable specification of model 20. Finally, we replicate the same set of four specifications in Columns (5)-(8) with the GPCP satellite measure. Year dummies are included in all regressions. Standard errors are clustered at the grid-cell level.[35]

As expected, there is a highly significant positive correlation between the gauge- and the satellite-based rainfall measures. Most important, all estimates of ξ are not significantly different from zero, with its point estimates switching sign across specifications. The hypothesis that ξ is negative due to the destruction of gauges in battlefields is strongly rejected, especially in specifications with grid-cell fixed effects, which are consistent with our panel specification where parameters are identified out of the variation over time in rainfall. In these columns, the point estimates of ξ are consistently positive and statistically insignificant. We conclude that there is no evidence that the gauge-based GPCC precipitation data are subject to non-classical measurement error in the DRC.

4.3 Measurement Error in Network Links

Another concern here is that the network may be measured with error. Recent research by Chandrasekhar and Lewis (2011) shows that regression of economic outcomes on network neighbors' outcomes, in the presence of measurement error of network links, can give rise to inconsistent estimates.[36] Moreover, the bias can work in different directions, and there is no general remedy to correct it.

In this section, we follow a Monte Carlo approach based on rewiring links in the observed network at random, and measuring the robustness of our estimates in such perturbed networks. We consider different assumptions about the extent and nature of measurement error of the network. More specifically, we postulate a data generating process, and then we introduce a specific (plausible) model of mis-measurement of network links. Then, we estimate the model as if the econometrician did not know the true network, but had to infer it from data measured with error. This procedure is generated for a large number of realizations of mis-measurement errors (1,000 draws per each case). For each Monte Carlo draw, we consider a data generating process based on the reduced form equation (14): The true parameters correspond to our baseline estimates, $\beta = 0.14$ and $\gamma = 0.09$, and the true network corresponds to the one used in our baseline estimation *after rewiring* a (sub-)set of alliances/enmities/neutral links according to a binomial process with a probability that is invariant across the draws. Then the benchmark 2SLS specification (Column 6, Table 2) is estimated on this fake dataset under the assumption that the network observed by the econometrician is identical to the baseline one *before* rewiring. Iterating over the Monte Carlo draws yields the sampling distribution of the estimates of β and γ subject to link measurement error.

The results are reported in Table 9. Each pair of columns refers to a particular mis-measurement probability, namely the probability governing the binomial re-wiring process. From left to right, we assume that the probability that links are measured with error is zero (no measurement error), 1%, 10%, 20%, 50% and one (pure noise), respectively. In all cases we report the mean and standard deviations of the sampling distribution based on 1,000 simulations.

[35] Recall that the GPCP satellite measure is only available at the 2.5×2.5 degree level, i.e., for larger cells than the two other measures that are at the 0.5×0.5 degree level. In this case, we cluster at the 2.5×2.5 cell level.

[36] It has been proven, however, that likelihood-based inference while ignoring the missing data mechanism leads to unbiased estimates under the assumption of missingness at random (MAR) (Little and Rubin, 2002). Mohan et al., (2013) provide conditions on the network for recoverability of parameters even when MAR is violated.

Table 8: Measurement error in rainfall.

	Dependent variable: GPCC gauge rainfall measure							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
model	linear		log-linear		linear		log-linear	
# ACLED conflict events	0.017	0.008	0.009	0.001	-0.069	0.016	-0.016	0.005
	(0.032)	(0.012)	(0.008)	(0.003)	(0.057)	(0.014)	(0.014)	(0.004)
TRMM satellite rainfall measure	0.639***	0.513***	0.714***	0.619***				
	(0.018)	(0.012)	(0.015)	(0.013)				
GPCP satellite rainfall measure					0.790***	1.073***	0.843***	1.233***
					(0.044)	(0.081)	(0.055)	(0.100)
(0.5 x 0.5) Grid Cell FE	No	Yes	No	Yes	No	Yes	No	Yes
Annual TE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9893	9893	9893	9893	9893	9893	9893	9893
R-squared	0.578	0.667	0.604	0.684	0.555	0.601	0.541	0.587

Notes: An observation is a given cell of resolution 0.5 x 0.5 degrees in a given year. The panel contains 761 cells covering DRC between 1998 and 2010. In Columns 3,4,5,6 all rainfall variables are in log. Robust standard errors are clustered at the (0.5 x 0.5) cell level in Columns 1-4 and at the (2.5 x 2.5) cell level in Columns 5-8. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

In the absence of measurement error – Columns 1 and 2 – we see that the mean of the sampling distribution is equal to the true values of the parameters. This confirms that our estimator is consistent. Consider, next, the pair of columns corresponding to interior probabilities of mis-measurement (1%, 10%, 20% and 50%). For instance, focus on the 10% measurement error case (fifth and sixth columns). In the first row (*enmity links only*), we draw from a distribution where each enmity link has a probability 10% to be in reality a neutral relationship (no link), whereas each neutral (no link) has a probability 10% to be in fact an enmity link. Since the number of neutrals exceeds by far that of enmities, this perturbation implies that in reality there are more enmity links than we observe. As expected, this experiment affects the estimate of γ more than it affects that of β. The estimate of β is close to the no measurement error benchmark (first column) at the three digit level, whereas the estimate of γ falls from 0.091 to 0.074. Namely, measurement error implies an attenuation bias. Next, consider the second row (*alliance links only*). This is the polar opposite scenario: alliance (rather than enmity) links are measured with error. Now, the estimate of β falls from 0.141 to 0.123, while the estimate of γ increases slightly. Finally, in the third row, we allow mis-measurement of both enmity and alliance links. In this case the estimate of β falls to 0.126 and that of γ falls to 0.08. A similar pattern is observed in the other interior columns. As one moves to the right, i.e., towards larger measurement errors, the attenuation bias becomes stronger. In the last two columns, when the econometrician observes pure noise for enmity (alliance) links, the estimate of the γ (β) tends to zero.

The lesson from this section is twofold. First, the Monte Carlo generated measurement errors in the links leads to an attenuation bias. This suggests that, under the plausible assumption that some information about existing links is missing, our regression analysis underestimates the spillover effects. Second, the extent of the bias is quantitatively modest. A measurement error of the order of 10% (which we regard as fairly large) yields an underestimate of the spillover parameters of the order of 11%-12%. Overall, this section confirms that our baseline estimates are robust to link measurement errors.

5 Policy Analysis

In this section, we perform some policy analysis. In particular, we perform two counterfactual exercises. First, we assess the contribution of each armed group to the conflict by performing a key player analysis as outlined in Section 2.4. The analysis has important policy implications. For instance, an international organization that aims at scaling down violence may be interested in the extent to which each of the combatant groups contributes to the conflict escalation. A naïve approach would be to target the groups involved in the largest number of fighting episodes. However, this would ignore the endogenous network effects that may be very important. Removing a group affects in a complicated way the incentives for the surviving groups to fight. For instance, if all agents had only first-order links, removing a group with many allies would not be useful, since its allies would increase their effort and offset the effect of the removal of the group. To the opposite, removing a group with many enemies would yield a large reduction in violence. Unfortunately, the calculation is far more complicated in the presence of a rich network structure, since the response of the surviving players depends also on higher-order links. Thus, the information about the structure of the network and the externality parameters is essential to guide the policy intervention.

The key player analysis is subject to some *caveat*. First, it is not clear how the international community could wipe out large armed groups unless at the cost of an unlikely large-scale military operation. Second, the removal of a large group may induce a reshuffling of alliances that our model is not well-suited to predict. With this in mind, in the second part of this section we study the effect

Table 9: Monte Carlo simulations testing link mismeasurement.

Probability of mismeasurement		0		0.01		0.1		0.2		0.5		1	
		β	γ	β	γ	β	γ	β	γ	β	γ	β	γ
Enmity links only	Mean	0.141	0.091	0.141	0.089	0.141	0.074	0.140	0.061	0.144	0.027	0.143	0.001
	S.D.	0.001	0.002	0.003	0.007	0.006	0.017	0.009	0.026	0.012	0.032	0.007	0.019
Alliance links only	Mean	0.141	0.091	0.138	0.092	0.123	0.096	0.103	0.097	0.044	0.098	0.001	0.091
	S.D.	0.001	0.001	0.004	0.013	0.016	0.025	0.025	0.028	0.036	0.032	0.009	0.010
Alliance & Enmity links	Mean	0.141	0.091	0.139	0.091	0.126	0.080	0.106	0.065	0.044	0.023	0.001	-0.001
	S.D.	0.001	0.002	0.005	0.015	0.018	0.028	0.029	0.035	0.042	0.043	0.017	0.021

Notes: This table reports the Mean and Standard Deviations of the Monte Carlo sampling distributions (1000 draws) of the 2SLS estimates of β and γ for different probabilities of link mismeasurement. The data generating process is based on true $\beta = 0.14$ and true $\gamma = 0.09$.

of pacification policies, where no armed group is removed but enmity links are selectively turned into neutral ones. We first consider an international peace process that simultaneously mutes all enmity links in DRC. Then, we analyze and rank (less ambitious) policies that pacify armed groups one by one.

5.1 Computation of the Counterfactual Equilibrium

Our analysis below requires us to compute counterfactual Nash equilibria corresponding to either the sequential removal of each fighting group (key player) or the rewiring of some enmity links (pacification policy).

We denote by G^b the benchmark network in which all groups fight. We set the externality parameters equal to $\beta = 0.1407$ and $\gamma = 0.0903$, in line with the point estimates of column (6) in Table 2.[37] We normalize V to unity. This entails no loss of generality, since changing V would simply rescale all players' payoffs and welfare. The equations (6) and (15)–(17) allow us to estimate e_i, the time-invariant unobserved heterogeneity. More formally:

$$\hat{e}_i = \mathrm{FE}_i - \Lambda^{\hat{\beta},\hat{\gamma}}(G)\left(1 - \Lambda^{\hat{\beta},\hat{\gamma}}(G)\right)\Gamma_i^{\hat{\beta},\hat{\gamma}}(G), \tag{22}$$

where $\Gamma_i^{\hat{\beta},\hat{\gamma}}(G) = 1/(1 + \hat{\beta}d_i^{+} - \hat{\gamma}d_i^{-})$ and $\Lambda^{\hat{\beta},\hat{\gamma}}(G) = 1 - 1/(\sum_j \Gamma_j^{\hat{\beta},\hat{\gamma}}(G))$. Consider, next, the vector of time-varying shifters $\mathbf{z}_{it}$ (rainfall, etc.). We perform the key player analysis in an average scenario in which all time-varying group-specific shifters are collapsed to their sample average, $\bar{\mathbf{z}}_i = \sum_{t=1998}^{2010} \frac{\mathbf{z}_{it}}{13}$, and denote by $\bar{\mathbf{Z}} = \{\bar{\mathbf{z}}_\mathbf{i}\}$ the estimated matrix of shifters. In other words, we compare an average year of conflict in the benchmark model to its corresponding counterfactual. Finally, we set the average of the time-varying i.i.d. unobserved shifter, $\sum_{t=1998}^{2010} \epsilon_{it}$, equal to zero for all groups.

We can then compute the Nash equilibrium. Following the analysis in Section 2.6, the vector of equilibrium fighting efforts is obtained by inverting the system of equilibrium conditions implied by equations (15) and (16). In matrix form, this yields:

$$\mathbf{x}^*(G^b) = (\mathbf{I} + \hat{\beta}\mathbf{A}^{+}(G^b) - \hat{\gamma}\mathbf{A}^{-}(G^b))^{-1}\left[\Lambda^{\hat{\beta},\hat{\gamma}}(G^b)(1 - \Lambda^{\hat{\beta},\hat{\gamma}}(G^b))\boldsymbol{\Gamma}^{\hat{\beta},\hat{\gamma}}(G^b) - (\bar{\mathbf{Z}}\widehat{\boldsymbol{\alpha}} + \widehat{\mathbf{e}})\right] \tag{23}$$

5.2 Key Player Analysis

To perform the key player analysis, we follow the procedure outlined above for each counterfactual equilibrium $G^b\backslash\{k\}$ (where $G^b\backslash\{k\}$ denotes the network after the removal of group k). The vector of equilibrium fighting efforts is given by equations which are analogous to equation (23) except that the dimension of the system is reduced by one, the adjacency matrix is $\mathbf{A}(G^b\backslash\{k\})$ and the parameters attached to the network structure are replaced by $\Lambda^{\hat{\beta},\hat{\gamma}}(G^b\backslash\{k\})$ and $\boldsymbol{\Gamma}^{\hat{\beta},\hat{\gamma}}(G^b\backslash\{k\})$. We compute rent dissipation before and after the removal of an agent k. Then, the change in the rent dissipation, which can be interpreted as the reduction in aggregate fighting, equals $\Delta\mathrm{RD}_k^{\hat{\beta},\hat{\gamma}} \equiv \mathrm{RD}^{\hat{\beta},\hat{\gamma}}\left(G^b\right) - \mathrm{RD}^{\hat{\beta},\hat{\gamma}}\left(G^b\backslash\{k\}\right)$, where $\mathrm{RD}^{\hat{\beta},\hat{\gamma}}\left(G^b\right) \equiv \sum_{i=1}^{n} x_i^*\left(G^b\right)$.

Figure 7 plots the percentage change in the rent dissipation, $\Delta\mathrm{RD}_k^{\hat{\beta},\hat{\gamma}}$ versus the observed share in total fighting $x_k^*\left(G^b\right)/\sum_{i=1}^{n} x_i^*\left(G^b\right)$ focusing on the 20 groups whose removal yields the largest

[37] All second-order conditions (cf. equation (3)) continue to hold for all groups in the counterfactual experiments in which one player is removed.

reduction in rent dissipation.[38] The Figure is in a log scale to ease the visualization of the main actors (red acronyms denote groups affiliated to foreign governments). Appendix Table 1 reports a complete ranking of the agents k according to the change in the rent dissipation $\Delta \mathrm{RD}_k^{\hat{\beta},\hat{\gamma}}$. The table also reports the contribution of each player to the total fighting when all groups are active.

Two findings are noteworthy. First, although the observed contribution of each group to total fighting correlates significantly with the reduction in total fighting associated with its removal, it is significantly below one.[39] For instance, the Rwanda-backed Rally for Congolese Democracy (Goma) is the single most active armed group in the DRC war (excluding the DRC government) accounting for ca. 8.6% of the total military activity in the DRC. However, its removal would reduce aggregate fighting by only 2.3%. In contrast, removing the FDLR, the main Hutu rebel group that fiercely opposes Tutsi influence in the region, would yield a reduction in violence of 13.8%, despite the fact that this group is responsible for less than 7% of the fighting activity. Some notorious (and virulent) rebel groups such as the LRA (Lord Resistance Army) and Laurent Nkunda's CNDP (National Congress for the Defense of People) are not among the top 20. Nor is the RCD-Kisangani, in spite of being the second most active group.

Second, foreign armies rank among the most disruptive players in the conflict. These include, in a ranked order, the armies of Rwanda, Uganda, Zimbabwe, Angola, South Africa, Sudan and Burundi (with Zambia and Chad also ranking among the top 30). It is remarkable that eight out of the twenty most virulent groups are foreign national armies. This confirms the anecdotal evidence that foreign intervention has been key for the escalation of the DRC war. To check the robustness of this result, we constructed a counterfactual in which all foreign troops are removed simultaneously. In this case, total fighting decreases by 24 percent!

Figure 8 shows the change in the rent dissipation as a function of the key player ranking. On the one hand, there are only 7 armed groups whose removal would reduce conflict by more than 2.5%. This shows how difficult it is to obtain large results by targeting individual players in a conflict with so many interconnected armies and militias. On the other hand, there are 5 groups whose removal would lead to a significant (2.5% or more) escalation rather than containment of the conflict.[40]

5.3 Pacification Policies

Using a similar approach, we study the effect of pacification policies aimed at reducing ethnic and political hostility across groups. As in the key player analysis, we study the change in rent dissipation associated with counterfactual scenarios. However, instead of removing some groups, we turn, selectively, some enmity links into neutral links. We start with a drastic (albeit unrealistic) counterfactual in which all enmity links are rewired into neutral relationships. The effect is very large: aggregate fighting is reduced by 54%. Not only enmities but also alliance links are important for the containment of the conflict: re-wiring all friendship links into neutral relationships yields increase in aggregate fighting by 95%. Thus, an even more dramatic counterfactual scenario is obtained by rewiring all enmity links and neutral relationships into friendship (i.e., alliance) links. The result is a reduction of aggregate violence in the order of 91% - almost full peace. This result

[38] We exclude from this ranking and alternative ones the DRC Army, since the counterfactual removal of the national army of the country is not an interesting scenario. The DRC army is active in both the benchmark and all counterfactual scenarios. We simply omit it from the figures and tables below.

[39] The correlation is between the two variables is 71%. The corresponding Spearman rank correlation is 90%.

[40] These groups are Mutiny of the Military Forces of DRC (-3.3%), Congolese Libration Movement (-5.2%), RCD-Kisangani (-5.4%), LRA (-7.5%), and CNDP (-8.8%).

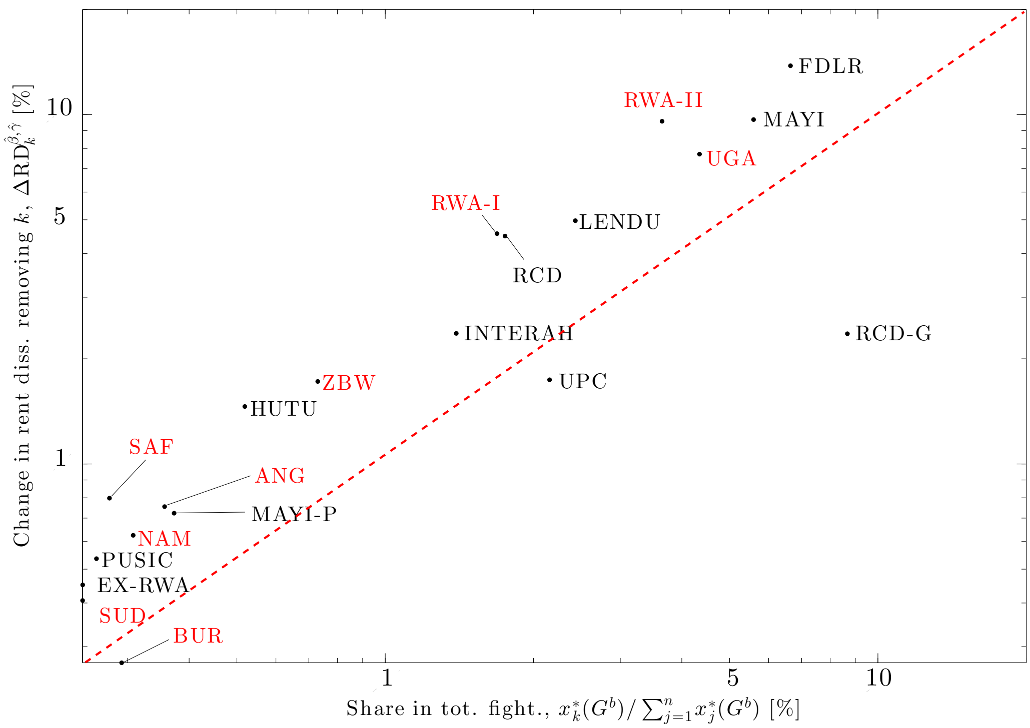


Figure 7: The figure reports the relative change in the rent dissipation associated with the removal of each of the top 20 groups in the ranking of the key player analysis versus the respective share in total fighting before removal. Both axes are in log-scale. The red dashed line indicates the 45 degree line. The acronyms stand for the following: FDLR = Democratic Forces for the Liberation of Rwanda, RWA-II = Mil. Forc. of Rwanda (2000-), MAYI = Mayi-Mayi Milita, UGA = Mil. Forc. of Uganda (1986-), LENDU = Lendu Ethnic Militia (DRC), RWA-I Mil. Forc. of Rwanda (1994-1999) RCD = Rally for Congolese Dem., INTERAH = Interahamwe Hutu Ethnic Militia, RCD-G = Rally for Congolese Dem. (Goma), UPC = Union of Congolese Patriots, ZBW = Mil. Forc. of Zimbabwe (1980-), HUTU = Hutu Rebels, SAF = Mil. Forc. of South Africa (1994-1999), ANG = FAA/MPLA = Mil. Forc. of Angola (1975-), MAYI-P = Mayi Mayi Militia(PARECO), NAM= Mil. Forc. of Namibia (1990-2005), PUSIC = Party for Unity and Safekeeping of Congo's Integrity, EX-RWA = Former Mil. Forc. of Rwanda,(1973-1994), SUD = Mil. Forc. of Sudan (1993-) and BUR = Mil. Forc. of Burundi (1996-2005). Group names indicated in red refer to government groups.

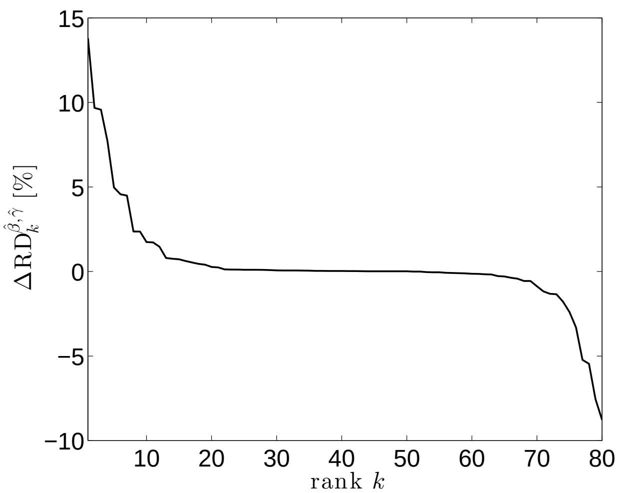


Figure 8: The figure shows the relative change in the rent dissipation associated with the removal of group k as a function of the key player ranking. Forces affiliated to the DRC army are not included in the figure.

echoes the insight of the theoretical analysis of the regular graph in Section 2.5.[41]

Since an intervention that wipes out all enmities in DRC would be utopistic, we consider a less ambitious targeted policy aimed to mute all enmity links associated with individual groups (i.e., we pacify one group at a time). Figure 9 shows the results (in logarithmic scale) focusing on the local militias whose pacification triggers a reduction in violence larger than 1%.[42] The largest effect is obtained from pacifying the RCD-Goma (17%), followed by FDLR (9.68%) and RCD-Kisangani (8.74%). As the figure shows, some of these groups are small players in terms of total contribution to violence. Appendix Table 2 reports the rent dissipation for all groups. Surprisingly, there are a few cases in which the pacification policy appears to be counterproductive: Ngiti Ethnic Militia, Alliance for Democratic Change, Alur Ethnic Militia.

6 Conclusion

In this paper, we construct a theory of conflict in which different groups compete over a fixed amount of resources. We introduce a network of alliances and enmities that we model as externalities added to a Tullock contest success function. Alliances are beneficial to each member, but are not unitary coalitions. Rather, each player acts strategically *vis-à-vis* both allies and enemies. We view our theory as especially useful in conflicts characterized by high fragmentation, non-transitive relations and decentralized military commands, all common features of civil conflicts.

We apply the theory to the analysis of the Second Congo War, one of the bloodiest civil conflicts in modern history. Our estimation of the network externalities is similar methodologically to that followed in the recent work of Acemoglu, Garcia-Jimeno and Robinson (2014) who tackle a reflection problem through an instrumental variable strategy. While they rely on historical information,

[41] The counterfactual scenarios above can be alternatively interpreted as a quantitative assessment of the mechanisms of our theory in explaining the data. Rewiring all alliances into neutral links is identical to setting the fighting spillover $\beta = 0$, whereas rewiring all enmities into neutral links is identical to setting $\gamma = 0$.

[42] The effect of pacifying foreign armies would be very large. However, the interpretation of this policy is ambiguous when applied to national armies. The reduction in rent dissipations associated with pacifying foreign armies are: Angola 77%, Namibia 21%, Rwanda 17%, Zimbabwe 15%, Chad 12%, Sudan 11%, Uganda 8%, South Africa 3%.

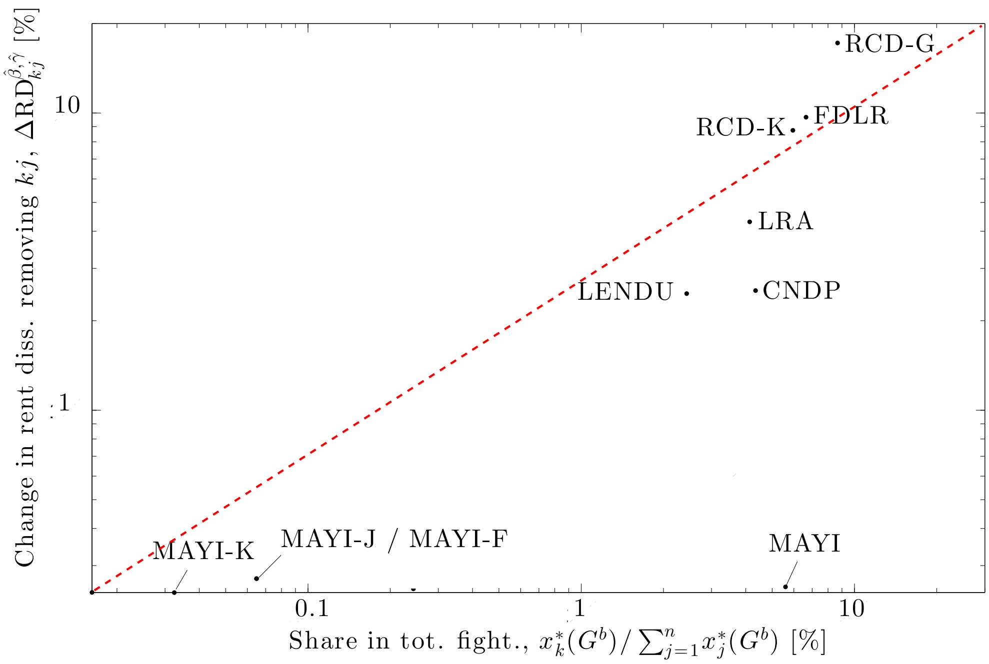


Figure 9: The figure reports the relative change in the rent dissipation associated with the pacification of enmity links with the DRC government of the top 10 local groups in the ranking of the key links analysis versus the respective share in total fighting before pacification. Both axes are in log-scale. The red dashed line indicates the 45 degrees line. The acronyms stand for the following: FDLR = Democratic Forces for the Liberation of Rwanda, MAYI = Mayi-Mayi Milita, LENDU = Lendu Ethnic Militia (DRC), LRA = Lord's Resistance Army, RCD-G = Rally for Congolese Dem. (Goma), RCD-K = Rally for Congolese Dem. (Kinsangani), CNDP = Nat. Congress for the Defense of the People, MAYI-K = Mayi-Mayi Militia (Kifuafua), MAYI-F = Mayi Mayi Milita (Cmdt La Fontaine) and MAYI-J = Mayi-Mayi Militia (Cmdt Jackson).

we exploit the exogenous variation over space and time in weather conditions. The signs of the estimated coefficients conform with the prediction of the theory. Each group's fighting effort is increasing in the total fighting of its enemies and decreasing in the total fighting of its allies. We then use our structural model to quantify the efficiency of various pacification policies. In particular we perform a key player analysis to identify which groups contribute most to the escalation of the conflict, either directly or indirectly, via the externalities they exercise on the other groups' fighting effort. The analysis highlight a number of interesting results, such as the key role of foreign armies in the conflict escalation.

The Congo War is a natural testing ground for our theory for being a conflict where most alliances and enmities are shallow links, and where many allied actors do not coordinate their actions. However, informal alliances and enmities and intransitive links are by no means unique to Congo. Rather, they are common fare in most modern civil conflicts, and pervasively so in for example the conflicts of Afghanistan, Somalia, Iraq, Sudan and Syria.

Even in the case of more conventional international wars, shallow links and intransitive links are not uncommon. For instance, the anti-Nazi alliance between the Soviet Union and the Anglo-Americans during World War II was a tactical alliance to defeat a common enemy. Well before the war was over, the Soviet Union and the Anglo-Americans were fighting strategically for conflicting objectives, each trying to secure the best political and military post-war outcome.[43] Another example is the intricate situation in the Balkans during WWII.[44] Similar considerations apply to earlier wars, from the Peloponnesian War in ancient Greece, to the Napoleonic Wars (Ke, Konrad, and Morath 2013), or to the alliances between warlords in China after the proclamation of the Republic in 1912.

Our analysis takes the first step towards understanding how webs of alliances and enmities can lead to escalation or containment of conflict. An important limitation is that we take the network as exogenous, and do not try to model its formation or dynamic evolution. Endogenizing network formation is a challenging extension that we leave to future research.

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[43] Several episodes corroborate this view. In August 1944, the Red Army refused to support the British-sponsored Polish Home Army during Warsaw Uprising. This tragic event was by-and-large a proxy war between two formally allied governments to gain control over Poland after the war.

[44] The Independent State of Croatia led by Ante Pavelic was sponsored by Nazi Germany, but was in poor terms with Italy, the main allied of Germany at the time, that occupied large sectors of Croatian Dalmatia. During the same period, Serbia under Milan Nedic was a Nazi puppet state collaborating with both Germany and Italy. The two sides – Croatian Ustasa and Serbian Chetniks – run a parallel ferocious ethnic war against each others (Goldstein 2013). Yet, the two enemies had a common enemy in Tito's partisan National Liberation Army.

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Appendix

Appendix A Proofs

The proof of Proposition 1 proceeds by first deriving the necessary first order conditions (FOCs), and then establishing that the second order conditions (SOCs) hold. For the latter, we provide a condition (cf. equation (3)) on the admissible parameters such that agent i's payoff, $\pi_i(\mathbf{x}^*_{-i}, x_i, G)$, given the other agents' efforts, $\mathbf{x}^*_{-i}$, is a locally concave function of x_i for all agents $i = 1, \ldots, n$. This implies that the necessary FOCs are also sufficient and hence pin down a local maximum for all agents.

Formally, this guarantees that the strategy profile $\mathbf{x}^*$ is a *local* Nash equilibrium, which is defined as follows:

Definition 2 (Alos-Ferrer and Ania, 2001)**.** *Consider a game with a finite number n of players, where the strategy spaces $\mathcal{S}_i$ are connected subsets of finite-dimensional Euclidean spaces, and the payoff functions $\pi_i : \mathcal{S} \to \mathbb{R}$ where $\mathcal{S} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_n$. A "local Nash equilibrium" for the game is a strategy profile $(x_i)_{i=1}^n \in \mathcal{S}$ such that, for all players i, x_i is a local maximum of $\pi_i(\cdot, \mathbf{x}_{-i})$, that is, there exists $\varepsilon > 0$ such that, for all x'_i in an ε-neighborhood of x_i, $\pi_i(x_i, \mathbf{x}_{-i}) \geq \pi_i(x'_i, \mathbf{x}_{-i})$.*

To ensure that this strategy profile $\mathbf{x}^*$ is, in addition, a Nash equilibrium in the standard sense, we must impose a lower bound $\underline{x}_i$ (cf. equation (36) in the proof of Proposition 1) on the strategy space of each agent to ensure that, given the other agents' efforts $\mathbf{x}^*_{-i}$, for agent i, x^*_i is a best reply over the entire admissible strategy space of i, given by $[\underline{x}_i, \infty)$. We then show that a *common* lower bound on the strategy spaces is simply defined by $\underline{x} \equiv \max_{i=1,\ldots,n} \underline{x}_i$. Under the condition $x_i \in [\underline{x}, \infty)$ for each agent i we also obtain that $\mathbf{x}^*$ is a Nash equilibrium.

Proof of Proposition 1. For our equilibrium characterization we start with the computation of the FOCs. With equation (2) we can write the payoff of agent i as follows

$$\begin{aligned}\pi_i(G, \mathbf{x}) &= V \frac{\varphi_i}{\sum_{j=1}^n \varphi_j} - x_i \\ &= V \frac{x_i + \beta \sum_{j=1}^n a^+_{ij} x_j - \gamma \sum_{j=1}^n a^-_{ij} x_j}{\sum_{j=1}^n \left(x_j + \beta \sum_{k=1}^n a^+_{jk} x_k - \gamma \sum_{k=1}^n a^-_{jk} x_k \right)} - x_i. \end{aligned} \tag{24}$$

The partial derivatives are given by

$$\begin{aligned}\frac{\partial \pi_i}{\partial x_i} &= V \frac{\frac{\partial \varphi_i}{\partial x_i} \sum_{j=1}^n \varphi_j - \varphi_i \sum_{j=1}^n \frac{\partial \varphi_j}{\partial x_i}}{\left(\sum_{j=1}^n \varphi_j \right)^2} - 1 \\ &= V \frac{\sum_{j=1}^n \varphi_j - \varphi_i (1 + \beta d^+_i - \gamma d^-_i)}{\left(\sum_{j=1}^n \varphi_j \right)^2} - 1, \end{aligned} \tag{25}$$

where we have used the fact that $\frac{\partial \varphi_j}{\partial x_i} = \delta_{ij} + \beta a^+_{ij} - \gamma a^-_{ij}$ and consequently $\sum_{j=1}^n \frac{\partial \varphi_j}{\partial x_i} = 1 + \beta d^+_i - \gamma d^-_i$. The FOCs are then given by $\frac{\partial \pi_i}{\partial x_i} = 0$. A simple manipulation of the FOC for each player i yields the individual operational performance

$$\varphi_i = \frac{1}{1 + \beta d^+_i - \gamma d^-_i} \left(1 - \frac{1}{V} \sum_{j=1}^n \varphi_j \right) \sum_{j=1}^n \varphi_j.$$

Summation over $i = 1, \ldots, n$ gives the aggregate operational performance

$$\sum_{i=1}^{n} \varphi_i = V \left(1 - \frac{1}{\sum_{i=1}^{n} \frac{1}{1+\beta d_i^+ - \gamma d_i^-}} \right).$$

In the following we denote by

$$\Gamma_i^{\beta,\gamma}(G) \equiv \frac{1}{1 + \beta d_i^+ - \gamma d_i^-},$$
$$\Lambda^{\beta,\gamma}(G) \equiv 1 - \frac{1}{\sum_{i=1}^{n} \frac{1}{1+\beta d_i^+ - \gamma d_i^-}} = 1 - \frac{1}{\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G)},$$

so that we can write aggregate operational performance at equilibrium as

$$\sum_{i=1}^{n} \varphi_i = V \Lambda^{\beta,\gamma}(G), \tag{26}$$

and individual operational performance at equilibrium as

$$\varphi_i = V \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G)) \Gamma_i^{\beta,\gamma}(G). \tag{27}$$

As shown below, by imposing restrictions such that the SOCs are satisfied (cf. equation (32)), we obtain as a result that that the operational performances in equation (27) are non-negative. Note that for the local hostility levels $\Gamma_i^{\beta,\gamma}(G) = \frac{1}{1+\beta d_i^+ - \gamma d_i^-}$ to be non-negative for all $i = 1, \ldots, n$, we need to require that $1 + \beta d_i^+ - \gamma d_i^- \geq 0$. This holds if $\gamma d_i^- \leq 1$, so that we require that $\gamma < 1/d_{\max}^-$. It further follows that $1 - \Lambda^{\beta,\gamma}(G) = \frac{1}{\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G)} > 0$ for a finite network (cf. Lemma 3). Next, we have that $\Lambda^{\beta,\gamma}(G) = 1 - \frac{1}{\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G)} > 0$ is equivalent to $\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G) > 1$. Observe that $\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G) = \sum_{i=1}^{n} \frac{1}{1+\beta d_i^+ - \gamma d_i^-} \geq \sum_{i=1}^{n} \frac{1}{1+\beta d_i^+} \geq \frac{n}{1+\beta d_{\max}^+}$. This is greater than one if $\beta < \frac{n-1}{d_{\max}^+}$, and this holds if $\beta < 1$. By requiring that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), \lambda_{\max}(G^-)\}$ we have that $\beta < 1$ for any non-empty graph, and $\varphi_i \geq 0$ for all $i = 1, \ldots, n$ in equation (27).

We next compute the equilibrium effort levels, based on the definition of operational performance (2) that we combine with equation (27). We have that

$$x_i + \beta \sum_{j=1}^{n} a_{ij}^+ x_j - \gamma \sum_{j=1}^{n} a_{ij}^- x_j = V \Lambda^{n,\beta}(G)(1 - \Lambda^{n,\beta}(G)) \Gamma_i^{\beta,\gamma}(G), \tag{28}$$

Denoting by $\mathbf{\Gamma}^{\beta,\gamma}(G) \equiv (\Gamma_1^{\beta,\gamma}(G), \ldots, \Gamma_n^{\beta,\gamma}(G))^\top$, we can write this in vector-matrix form as

$$(\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)\mathbf{x} = V \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G)) \mathbf{\Gamma}^{\beta,\gamma}(G) \tag{29}$$

We now proceed by deriving sufficient conditions such that the matrix $\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-$ is invertible. Provided that the SOCs hold (see below), this also guarantees the existence and uniqueness of the Nash equilibrium. Observe that the matrix $\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-$ is invertible if its determinant is not zero. We first consider two special cases. First, assume that $\gamma = 0$. Then the inverse $(\mathbf{I}_n + \beta \mathbf{A})^{-1}$ exists only if $|\mathbf{I}_n + \beta \mathbf{A}| \neq 0$. We have that $|\mathbf{I}_n + \beta \mathbf{A}| = 0$ if and only if $-\frac{1}{\beta}$ is an eigenvalue of $\mathbf{A}$. If the smallest eigenvalue $\lambda_{\min}$ is such that $\beta < 1/|\lambda_{\min}|$, then this condition is satisfied. This is because $|\lambda_{\min}| < \frac{1}{\beta}$ implies that $-\frac{1}{\beta} < \lambda_{\min}$ and we must have that $-\frac{1}{\beta} \neq \lambda_i$ for all $i = 1, \ldots, n$. Second, assume that $\beta = 0$. Then the matrix $\mathbf{I}_n - \gamma \mathbf{A}^-$ is invertible if $\gamma < 1/\lambda_{\max}(G^-)$, where $\lambda_{\max}(G^-)$ is the largest real eigenvalue of G^-. In the general case a sufficient condition for the matrix $\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-$ to be invertible, is that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), \lambda_{\max}(G^-)\}$. This is because the determinant of a matrix of the form $\mathbf{I}_n - \sum_{j=1}^{p} \lambda_j \mathbf{W}_j$ is strictly positive if $\sum_{j=1}^{p} |\lambda_j| < 1/\max_{j=1,\ldots,p} \|\mathbf{W}_j\|$, where $\|\mathbf{W}_j\|$ is any matrix norm, including the spectral norm, which corresponds to the largest eigenvalue of $\mathbf{W}_j$ (cf. e.g. Lee and Liu, 2010). A strictly positive determinant then implies invertibility. Further, note that $\lambda_{\max}(G^-) < d^-_{\max}$ (Cvetkovic, Doob and Sachs 1995), so that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), d^-_{\max}\}$ implies that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), \lambda_{\max}(G^-)\}$. From our previous discussion we see that this also guarantees that the operational performances in equation (27) are non-negative.

In the following we provide an explicit computation of the equilibrium fighting efforts when invertibility holds. More precisely, when the matrix $\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-$ is invertible, we obtain from (29) the equilibrium effort levels,

$$\mathbf{x}^* = V \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G)) \underbrace{(\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)^{-1} \mathbf{\Gamma}^{\beta,\gamma}(G)}_{\mathbf{c}^{\beta,\gamma}(G)},$$

where we have introduced the centrality

$$\mathbf{c}^{\beta,\gamma}(G) \equiv (\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)^{-1} \mathbf{\Gamma}^{\beta,\gamma}(G).$$

We can write the equilibrium effort for each agent i as follows

$$x_i^* = V \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G)) c_i^{\beta,\gamma}(G). \tag{30}$$

This is exactly equation (8) in the proposition. Moreover, equilibrium payoff is given by

$$\pi_i^*(G) \equiv \pi_i(\mathbf{x}^*, G) = V \frac{\varphi_i^*(G)}{\sum_{j=1}^{n} \varphi_j^*(G)} - x_i^* = V(1 - \Lambda^{\beta,\gamma}(G)) \left(\Gamma_i^{\beta,\gamma}(G) - \Lambda^{\beta,\gamma}(G) c_i^{\beta,\gamma}(G) \right),$$

and we obtain equation (9) in the proposition.

In order to show that the necessary first order conditions are also sufficient, we next compute the second order conditions (SOCs). From equation (25) we have that the second partial cross derivative of agent i's payoff is given by

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial x_i \partial x_j} = \frac{V}{\left(\sum_{k=1}^{n} \varphi_k\right)^4} & \left[\left(\sum_{k=1}^{n} \varphi_k \right)^2 \left(\sum_{k=1}^{n} \frac{\partial \varphi_k}{\partial x_j} - (1 + \beta d_i^+ - \gamma d_i^-) \frac{\partial \varphi_i}{\partial x_j} \right) \right. \\ & \left. -2 \left(\sum_{k=1}^{n} \varphi_k \right) \left(\sum_{k=1}^{n} \varphi_k - \varphi_i (1 + \beta d_i^+ - \gamma d_i^-) \right) \sum_{k=1}^{n} \frac{\partial \varphi_k}{\partial x_j} \right]. \end{aligned}$$

Using the fact that

$$\sum_{k=1}^{n} \frac{\partial \varphi_k}{\partial x_j} = 1 + \beta d_j^+ - \gamma d_j^-,$$

and that

$$\begin{aligned} \Gamma_i^{\beta,\gamma}(G) &= \frac{1}{1 + \beta d_i^+ - \gamma d_i^-}, \\ \sum_{k=1}^{n} \varphi_k &= V \Lambda^{\beta,\gamma}(G) = V \frac{1}{\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G)} \left(\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G) - 1 \right) = V \frac{\mathbf{u}^\top \mathbf{\Gamma}^{\beta,\gamma}(G) - 1}{\mathbf{u}^\top \mathbf{\Gamma}^{\beta,\gamma}(G)}, \end{aligned}$$

we can write the second partial cross derivative as

$$\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} = \frac{1}{V\Lambda^{\beta,\gamma}(G)^2}\left[\frac{1-2\Lambda^{\beta,\gamma}(G)}{\Gamma_j^{\beta,\gamma}(G)} - \frac{\delta_{ij}+\beta\mathbb{1}_{\{j\in\mathcal{N}_i^+\}}-\gamma\mathbb{1}_{\{j\in\mathcal{N}_i^-\}}}{\Gamma_i^{\beta,\gamma}(G)}\right]. \tag{31}$$

From equation (31) we find that

$$\frac{\partial^2 \pi_i}{\partial x_i^2} = -\frac{2}{V\Lambda^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G)} = -\frac{2}{V}\frac{\sum_{j=1}^n \Gamma_j^{\beta,\gamma}(G)}{\sum_{j=1}^n \Gamma_j^{\beta,\gamma}(G)-1}(1+\beta d_i^+ - \gamma d_i^-), \tag{32}$$

which is negative if $\Lambda^{\beta,\gamma}(G) > 0$ and $1+\beta d_i^+ - \gamma d_i^- > 0$. The last inequality requires that the local hostility level of agent i is positive, i.e. $\Gamma_i^{\beta,\gamma}(G) > 0$. It then follows that the equilibrium payoff function is locally concave under the condition in equation (3).

In the remainder of the proof we show that the equilibrium payoff function is globally concave over a specified domain, by bounding the effort levels (from below). This implies that the marginal payoff of an agent from deviating from the Nash equilibrium strategy is always negative. Observe that

$$\begin{aligned}
\pi_i(\mathbf{x}_{-i}^*, x_i, G) &= V\frac{\varphi_i(\mathbf{x}_{-i}^*, x, G)}{\sum_{j=1}^n \varphi_j(\mathbf{x}_{-i}^*, x, G)} - x_i \\
&= V\frac{\varphi_i(\mathbf{x}^*, G) - (x_i^* - x_i)}{\sum_{j=1}^n \varphi_j(\mathbf{x}^*, G) - (1+\beta d_i^+ - \gamma d_i^-)(x_i^* - x_i)} - x_i \\
&= V\frac{A_i^{\beta,\gamma}(G) + \Gamma_i^{\beta,\gamma}(G)x_i}{B_i^{\beta,\gamma}(G)+x_i} - x_i,
\end{aligned} \tag{33}$$

where we have denoted by

$$\begin{aligned}
&A_i^{\beta,\gamma}(G) \equiv \Gamma_i^{\beta,\gamma}(G)(\varphi_i^* - x_i^*) = V\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))\Gamma_i^{\beta,\gamma}(G)\left(\Gamma_i^{\beta,\gamma}(G) - c_i^{\beta,\gamma}(G)\right), \\
&B_i^{\beta,\gamma}(G) \equiv \Gamma_i^{\beta,\gamma}(G)\sum_{j=1}^n \varphi_j^* - x_i^* = V\Lambda^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) - x_i^* = V\Lambda^{\beta,\gamma}(G)\left(\Gamma_i^{\beta,\gamma}(G) - (1-\Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G)\right),
\end{aligned}$$

and the centrality $c_i^{\beta,\gamma}(G)$ has been defined in equation (7). We first assume that $B_i^{\beta,\gamma}(G) > 0$. Figure I illustrates the function $\pi_i(\mathbf{x}_{-i}^*, x, G)$ in equation (33) for different values of x_i. For $x_i > -B_i^{\beta,\gamma}(G)$ it is concave with a maximum at x_i^*, and decreasing for values of $x_i > x_i^*$ with an asymptote given by $-x_i$.[45] In particular, if x_i is larger than the discontinuity at $-B_i^{\beta,\gamma}(G)$ of $\pi_i(\mathbf{x}_{-i}^*, x, G)$ then we have that $\pi_i(\mathbf{x}_{-i}^*, x_i, G) < \pi_i(\mathbf{x}^*, G)$. Note that $\pi_i(\mathbf{x}_{-i}^*, x_i, G) < \pi_i(\mathbf{x}^*, G)$ iff

$$\begin{aligned}
V\frac{A_i^{\beta,\gamma}(G) + \Gamma_i^{\beta,\gamma}(G)x_i}{B_i^{\beta,\gamma}(G)+x_i} - x_i < \pi_i^* &= V\frac{\varphi_i^*}{\sum_{j=1}^n \varphi_j^*(G)} - x_i^* \\
&= V(1-\Lambda^{\beta,\gamma}(G))\Gamma_i^{\beta,\gamma}(G) - x_i^* \\
&= V(1-\Lambda^{\beta,\gamma}(G))\left(\Gamma_i^{\beta,\gamma}(G) - \Lambda^{\beta,\gamma}(G)c_i^{\beta,\gamma}(G)\right),
\end{aligned} \tag{34}$$

[45] Note that $\frac{\partial^2 \pi_i(\mathbf{x}_{-i}^*, x_i, G)}{\partial x_i^2} = -\frac{2V\left(B_i^{\beta,\gamma}(G) - A_i^{\beta,\gamma}(G)/\Gamma_i^{\beta,\gamma}(G)\right)\Gamma_i^{\beta,\gamma}(G)}{(B_i^{\beta,\gamma}(G)+x_i)^3} < 0$ for $x_i > -B_i^{\beta,\gamma}(G)$ and using the fact that $B_i^{\beta,\gamma}(G) - A_i^{\beta,\gamma}(G)/\Gamma_i^{\beta,\gamma}(G) = \Gamma_i^{\beta,\gamma}(G)\sum_j \varphi_j^* - x_i^* - \varphi_i^* + x_i^* = V\Gamma_i^{\beta,\gamma}(G)\Lambda^{\beta,\gamma}(G) - \varphi_i^* = V\Lambda^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) - V\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))\Gamma_i^{\beta,\gamma}(G) = V\Lambda^{\beta,\gamma}(G)^2\Gamma_i^{\beta,\gamma}(G) > 0$.

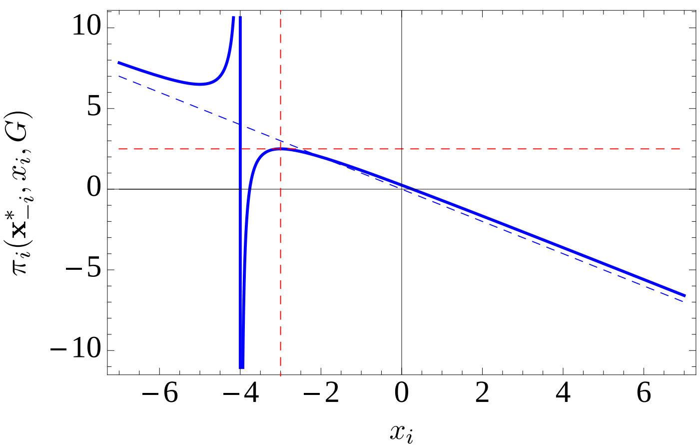

Figure I: The payoff function $\pi_i(\mathbf{x}^*_{-i}, x_i, G)$ from equation (33) for different values of x_i. The horizontal red dashed line indicates π_i^*, the red vertical dashed line indicates x_i^*, while the blue vertical solid line indicates the point of discontinuity, $-B_i^{\beta,\gamma}(G)$. If x_i is larger than the discontinuity at $-B_i^{\beta,\gamma}(G)$ then we have that $\pi_i(\mathbf{x}^*_{-i}, x_i, G) < \pi_i(\mathbf{x}^*, G)$. The blue dashed line indicates the asymptote with slope -1.

where we have denoted by $\pi_i^* \equiv \pi_i(\mathbf{x}^*, G)$. Next, observe from the FOC that

$$\frac{\partial \pi_i(\mathbf{x}^*_{-i}, x_i, G)}{\partial x_i} = -\frac{A_i^{\beta,\gamma}(G) - B_i^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) + (B_i^{\beta,\gamma}(G) + x_i)^2}{(B_i^{\beta,\gamma}(G) + x_i)^2} = 0, \tag{35}$$

it follows that[46]

$$x_i = -B_i^{\beta,\gamma}(G) + \sqrt{B_i^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) - A_i^{\beta,\gamma}(G)} = x_i^*,$$

and inserting into $\pi_i(\mathbf{x}^*_{-i}, x_i, G)$ yields π_i^*. We then define

$$\underline{x}_i \equiv -B_i^{\beta,\gamma}(G). \tag{36}$$

Assuming that $B_i^{\beta,\gamma}(G) \geq 0$ we get $\underline{x}_i < 0$. Note that if $B_i^{\beta,\gamma}(G) > 0$ then the sign of $A_i^{\beta,\gamma}(G)$ does not qualitatively change the functional form of $\pi_i(\mathbf{x}^*_{-i}, x, G)$ (more precisely, $\pi_i(\mathbf{x}^*_{-i}, x, G)$ is a concave function for $x_i \geq -B_i^{\beta,\gamma}(G)$ with a global maximum over its domain at x_i^*; see also footnote 45). In contrast, if $B_i^{\beta,\gamma}(G) < 0$ then we must have that $A_i^{\beta,\gamma}(G)$ must be negative as well. More precisely, if $B_i^{\beta,\gamma}(G) < 0$ then we must have that $A_i^{\beta,\gamma}(G) < B_i^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G)$. Otherwise we would have that $\frac{\partial \pi_i(\mathbf{x}^*_{-i}, x_i, G)}{\partial x_i} < 0$ (see equation (35)), violating the FOC. However, under this condition, the functional form of $\pi_i(\mathbf{x}^*_{-i}, x, G)$ remains qualitatively unchanged.

To summarize, for all the above cases we have that $\pi_i(\mathbf{x}^*_{-i}, x_i, G) - \pi_i(\mathbf{x}^*, G) < 0$ if $x_i \in [\underline{x}_i, \infty)$. Hence, this characterizes a Nash equilibrium. This completes the proof. ∎

Remark 1. *It is possible to compute a common lower bound in Proposition 1, $\underline{x}$, on the effort levels x_i, such that $\pi_i(\mathbf{x}^*_{-i}, x_i, G) - \pi_i(\mathbf{x}^*, G) < 0$ holds for all $i = 1, \ldots, n$ if $\mathbf{x} \in [\underline{x}, \infty)^n$. Moreover,*

[46] W.l.o.g. set $V = 1$. Then, using the fact that $A_i^{\beta,\gamma}(G) = \Gamma_i^{\beta,\gamma}(G)\Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))(\Gamma_i^{\beta,\gamma}(G) - c_i^{\beta,\gamma}(G))$, $B_i^{\beta,\gamma}(G) = \Lambda^{\beta,\gamma}(G)(\Gamma_i^{\beta,\gamma}(G) - (1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G))$ and $x_i^* = \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G)$, we find that $-B_i^{\beta,\gamma}(G) + \sqrt{B_i^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) - A_i^{\beta,\gamma}(G)} = \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G) = x_i^*$.

we also require that $\underline{x} < x_i^*$ *for all* $i = 1, \ldots, n$. *We denote this common lower bound as* $\underline{x} \equiv \max_{i=1,\ldots,n} \underline{x}_i$, *and we require that* $\forall i = 1, \ldots, n$

$$\underline{x} \leq x_i^* = V\Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G). \tag{37}$$

Note that equation (37) is equivalent to

$$\max_{i=1,\ldots,n} \left\{ -B_i^{\beta,\gamma}(G) \right\} = \max_{i=1,\ldots,n} \left\{ x_i^* - \Gamma_i^{\beta,\gamma}(G) \sum_{j=1}^{n} \varphi_j^* \right\} \leq \min_{i=1,\ldots,n} x_i^*,$$

or

$$\max_{i=1,\ldots,n} \left\{ c_i^{\beta,\gamma}(G) - \frac{\Gamma_i^{\beta,\gamma}(G)}{1 - \Lambda^{\beta,\gamma}(G)} \right\} \leq \min_{i=1,\ldots,n} c_i^{\beta,\gamma}(G),$$

Note that this is equivalent to

$$\max_{i=1,\ldots,n} \underbrace{\Gamma_i^{\beta,\gamma}(G)}_{\geq 0 \text{ by Prop. } 1} \underbrace{\left(c_i^{\beta,\gamma}(G)/\Gamma_i^{\beta,\gamma}(G) - \sum_{j=1}^{n} \Gamma_j^{\beta,\gamma}(G) \right)}_{\leq 0 \text{ if } B_i^{\beta,\gamma}(G) \geq 0} \leq \min_{i=1,\ldots,n} c_i^{\beta,\gamma}(G) \tag{38}$$

Proof of Lemma 1. The proof of the lemma builds on a first order Taylor approximation in β and γ of the centrality $\mathbf{c}^{\beta,\gamma}(G)$ defined in equation (7). Using the fact that $\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^- = (\mathbf{I}_n + \beta\mathbf{A}^+)(\mathbf{I}_n - \gamma\mathbf{A}^-) + \beta\gamma\mathbf{A}^+\mathbf{A}^- = (\mathbf{I}_n + \beta\mathbf{A}^+)(\mathbf{I}_n - \gamma\mathbf{A}^-) + O(\beta\gamma)$, and $[(\mathbf{I}_n + \beta\mathbf{A}^+)(\mathbf{I}_n - \gamma\mathbf{A}^-)]^{-1} = (\mathbf{I}_n - \gamma\mathbf{A}^-)^{-1}(\mathbf{I}_n + \beta\mathbf{A}^+)^{-1}$, where $(\mathbf{I}_n - \gamma\mathbf{A}^-)^{-1} = \sum_{k=0}^{\infty} \gamma^k(\mathbf{A}^-)^k$ and $(\mathbf{I}_n + \beta\mathbf{A}^+)^{-1} = \sum_{k=0}^{\infty}(-1)^k\beta^k(\mathbf{A}^+)^k$, we can write

$$\begin{aligned}
(\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-)^{-1} &= (\mathbf{I}_n - \gamma\mathbf{A}^-)^{-1}(\mathbf{I}_n + \beta\mathbf{A}^+)^{-1} + O(\beta\gamma) \\
&= \left(\mathbf{I}_n + \gamma\mathbf{A}^- + \sum_{k=2}^{\infty} \gamma^k(\mathbf{A}^-)^k \right) \left(\mathbf{I}_n - \beta\mathbf{A}^+ + \sum_{k=2}^{\infty}(-1)^k\beta^k(\mathbf{A}^+)^k \right) + O(\beta\gamma) \\
&= \mathbf{I}_n + \gamma\mathbf{A}^- - \beta\mathbf{A}^+ + \sum_{k=2}^{\infty} \gamma^k(\mathbf{A}^-)^k + \sum_{k=2}^{\infty}(-1)^k\beta^k(\mathbf{A}^+)^k + \gamma\mathbf{A}^- \sum_{k=2}^{\infty}(-1)^k\beta^k(\mathbf{A}^+)^k \\
&+ \sum_{k=2}^{\infty} \gamma^k(\mathbf{A}^-)^k\beta\mathbf{A}^+ + \left(\sum_{k=2}^{\infty} \gamma^k(\mathbf{A}^-)^k \right) \left(\sum_{k=2}^{\infty}(-1)^k\beta^k(\mathbf{A}^+)^k \right) + O(\beta\gamma) \\
&= \mathbf{I}_n + \sum_{k=1}^{\infty} \gamma^k(\mathbf{A}^-)^k + \sum_{k=1}^{\infty}(-1)^k\beta^k(\mathbf{A}^+)^k + O(\beta\gamma).
\end{aligned} \tag{39}$$

Hence, in leading order in β and γ the centrality in equation (7) can then be written as follows

$$\begin{aligned}
\mathbf{c}^{\beta,\gamma}(G) &= \left(\sum_{k=0}^{\infty} \gamma^k(\mathbf{A}^-)^k + \sum_{k=0}^{\infty}(-1)^k\beta^k(\mathbf{A}^+)^k - \mathbf{I}_n \right) \mathbf{\Gamma}^{\beta,\gamma}(G) + O(\beta\gamma) \\
&= \mathbf{b}_{\mathbf{\Gamma}^{\beta,\gamma}(G)}(\gamma, G^-) + \mathbf{b}_{\mathbf{\Gamma}^{\beta,\gamma}(G)}(-\beta, G^+) - \mathbf{\Gamma}^{\beta,\gamma}(G) + O(\beta\gamma).
\end{aligned}$$

This completes the proof. ∎

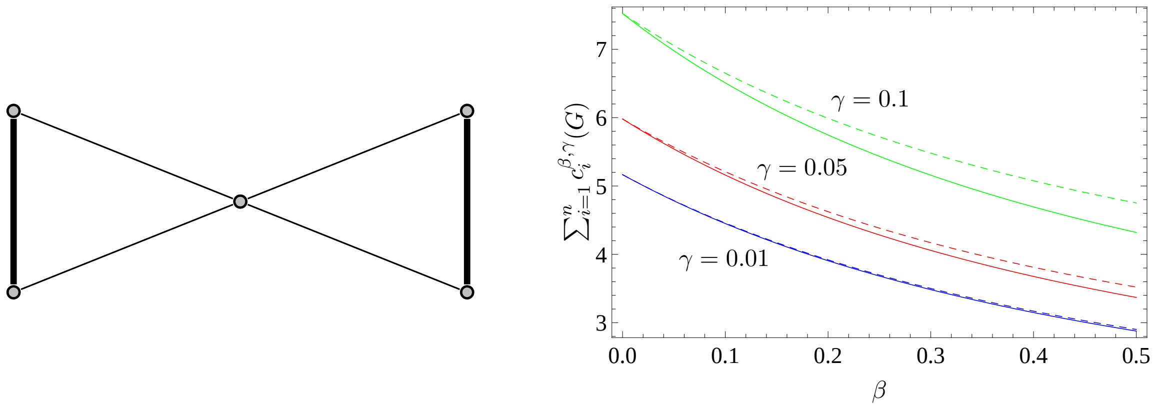

Figure II: (Left panel) Illustration of a bowtie graph, where the central agent is in conflict with all other agents and the peripheral pairs of agents are allied. Alliances are indicated with thick lines while conflict relationships are indicated with thin lines. (Right panel) The first-order approximation (dashed lines) used to derive the equilibrium efforts in equation (40), and the exact value (solid lines) for the total centrality $\sum_{i=1}^{n} c_i^{\beta,\gamma}(G)$ for different values of β and γ in the bowtie graph.

Proof of Lemma 2. The proof of the lemma builds on a first order Taylor approximation in β and γ of the equilibrium fighting efforts $x_i^*(G)$ in equation (8) and payoffs $\pi_i^*(G)$ from equation (9). First, observe that for $\beta \to 0$ and $\gamma \to 0$ we have that

$$\Gamma_i^{\beta,\gamma}(G) = \frac{1}{1+\beta d_i^+ - \gamma d_i^-} = 1 - \beta d_i^+ + \gamma d_i^- + O\left(\beta^2\right) + O\left(\gamma^2\right) + O\left(\beta\gamma\right),$$

and

$$1 - \Lambda^{\beta,\gamma}(G) = \frac{1}{\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G)} = \frac{1}{n} + \frac{2}{n^2}\left(\beta m^+ - \gamma m^-\right) + O\left(\beta^2\right) + O\left(\gamma^2\right) + O\left(\beta\gamma\right),$$

where we have denoted by $m^+ = \frac{1}{2}\sum_{i=1}^{n} d_i^+$ and $m^- = \frac{1}{2}\sum_{i=1}^{n} d_i^-$. Also, we have that

$$\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G)) = \frac{n-1}{n^2} - \frac{2m^-(n-2)\gamma}{n^3} + \frac{2m^+(n-2)\beta}{n^3} + O\left(\beta^2\right) + O\left(\gamma^2\right) + O\left(\beta\gamma\right).$$

Moreover, from equation (39) in the proof of Lemma 1 we have that

$$\begin{aligned}
(\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-)^{-1} &= (\mathbf{I}_n - \gamma\mathbf{A}^-)^{-1}(\mathbf{I}_n + \beta\mathbf{A}^+)^{-1} + O\left(\beta\gamma\right) \\
&= (\mathbf{I}_n + \gamma\mathbf{A}^-)(\mathbf{I}_n - \beta\mathbf{A}^+)(\mathbf{u} + \gamma\mathbf{d}^+ - \beta\mathbf{d}^- + O\left(\beta^2\right) + O\left(\gamma^2\right) + O\left(\beta\gamma\right) \\
&= \mathbf{I}_n + \gamma\mathbf{A}^- - \beta\mathbf{A}^+ + O\left(\beta^2\right) + O\left(\gamma^2\right) + O\left(\beta\gamma\right).
\end{aligned}$$

It then follows that

$$\begin{aligned}
(\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-)^{-1}\mathbf{\Gamma}^{\beta,\gamma}(G) &= (\mathbf{I}_n + \gamma\mathbf{A}^- - \beta\mathbf{A}^+)(\mathbf{u} + \gamma\mathbf{d}^- - \beta\mathbf{d}^+) + O\left(\beta^2\right) + O\left(\gamma^2\right) + O\left(\beta\gamma\right) \\
&= \mathbf{u} + 2\gamma\mathbf{d}^- - 2\beta\mathbf{d}^+ + O\left(\beta^2\right) + O\left(\gamma^2\right) + O\left(\beta\gamma\right),
\end{aligned}$$

and we get for the equilibrium fighting efforts $x_i^*(G)$ that

$$\begin{aligned}
x_i^*(G) &= V\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))((\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-)^{-1}\mathbf{\Gamma}^{\beta,\gamma}(G))_i \\
&= V\left(\frac{n-1}{n^2} - \frac{2m^-(n-2)\gamma}{n^3} + \frac{2m^+(n-2)\beta}{n^3}\right)(1 + 2\gamma d_i^- - 2\beta d_i^+) + O\left(\beta\gamma\right) \\
&= V\left(\frac{n-1}{n^2} - \frac{2(n-2)m^- - 2n(n-1)d_i^-}{n^3}\gamma + \frac{2(n-2)m^+ - 2n(n-1)d_i^+}{n^3}\beta\right) + O\left(\beta\gamma\right).
\end{aligned}$$

Denoting by $A^{\beta,\gamma}(G) \equiv \frac{n-1}{n^2} + \beta\frac{2(n-2)}{n^3}m^+ - \gamma\frac{2(n-2)}{n^3}m^-$ and $B \equiv \frac{2(n-1)}{n^2}$, we can write this as follows

$$x_i^*(G) = V\left(A^{\beta,\gamma}(G) - B\left(\beta d_i^+ - \gamma d_i^-\right)\right) + O\left(\beta\gamma\right). \tag{40}$$

Next, note that the payoff of agent i in equilibrium is given by $\pi_i^*(G) = V(1-\Lambda^{\beta,\gamma}(G))\Gamma_i^{\beta,\gamma}(G) - x_i^*(G)$. Using the equilibrium effort from above it then follows that the equilibrium payoff of agent i is given by

$$\begin{aligned}
\pi_i^*(G) &= V(1-\Lambda^{\beta,\gamma}(G))\Gamma_i^{\beta,\gamma}(G) - x_i^*(G) \\
&= \left(\frac{1}{n} - \frac{2\gamma m^- - 2\beta m^+}{n^2}\right)(1+\gamma d_i^- - \beta d_i^+) - x_i^*(G) + O\left(\beta\gamma\right) \\
&= V\left(\frac{1}{n^2} + \frac{4m^+ + n(n-2)d_i^+}{n^3}\beta - \frac{4m^- + n(n-2)d_i^-}{n^3}\gamma\right) + O\left(\beta\gamma\right).
\end{aligned}$$

Denoting by $C^{\beta,\gamma}(G) \equiv \frac{1}{n^2} + \beta\frac{4}{n^3}m^+ - \gamma\frac{4}{n^3}m^-$ and $D \equiv \frac{n-2}{n^2}$, this can be written as follows

$$\pi_i^*(G) = V\left(C^{\beta,\gamma}(G) + D\left(\beta d_i^+ - \gamma d_i^-\right)\right) + O\left(\beta\gamma\right).$$

This completes the proof. ∎

Figure II shows an illustration of the first-order approximation used to derive the equilibrium efforts in equation (40), and the exact value for the total centrality $\sum_{i=1}^n c_i^{\beta,\gamma}(G)$ for different values of β and γ for a bowtie graph.

Online Appendix B Proofs and Technical Analysis

In the following lemma we state additional results on the monotonicity and bounds on the local hostility levels and the total local fighting intensity.

Lemma 3. *Both the local hostility levels $\Gamma_i^{\beta,\gamma}(G)$ and the total local fighting intensity $\Lambda^{\beta,\gamma}(G)$ are decreasing with β, and increasing with γ. Moreover if $\gamma < 1/d^-_{\max}$ then $\Gamma_i^{\beta,\gamma}(G) \geq 0$, and we have that $0 \leq \Lambda^{\beta,0}(G) \leq \Lambda^{\beta,\gamma}(G) \leq \Lambda^{0,\gamma}(G) \leq 1$.*

Proof of Lemma 3. We first analyze changes in the local hostility levels with chaining the parameters β and γ. Note that

$$\frac{\partial \Gamma_i^{\beta,\gamma}(G)}{\partial \beta} = -\frac{d_i^+}{(1+\beta d_i^+ - \gamma d_i -)^2} = -d_i^+ \Gamma_i^{\beta,\gamma}(G)^2 \leq 0,$$

and

$$\frac{\partial \Gamma_i^{\beta,\gamma}(G)}{\partial \gamma} = \frac{d_i^-}{(1+\beta d_i^+ - \gamma d_i^-)^2} = d_i^- \Gamma_i^{\beta,\gamma}(G)^2 \geq 0,$$

Similarly, we find that

$$\frac{\partial \Lambda^{\beta,\gamma}(G)}{\partial \beta} = -\frac{\sum_{i=1}^n d_i^+ \Gamma_i^{\beta,\gamma}(G)^2}{\left(\sum_{i=1}^n \Gamma_i^{\beta,\gamma}(G)\right)^2} \leq 0,$$

and

$$\frac{\partial \Lambda^{\beta,\gamma}(G)}{\partial \gamma} = \frac{\sum_{i=1}^n d_i^- \Gamma_i^{\beta,\gamma}(G)^2}{\left(\sum_{i=1}^n \Gamma_i^{\beta,\gamma}(G)\right)^2} \geq 0.$$

Hence, both $\Gamma_i^{\beta,\gamma}(G)$ and $\Lambda^{\beta,\gamma}(G)$ are decreasing with β, and increasing with γ. Hence, it follows that

$$\Lambda^{\beta,0}(G) \leq \Lambda^{\beta,\gamma}(G) \leq \Lambda^{0,\gamma}(G).$$

We further have that

$$\Gamma_i^{\beta,0}(G) \to \begin{cases} 1, & \text{as } \beta \to 0, \\ \frac{1}{1+d_i^+}, & \text{as } \beta \to 1, \end{cases}$$

and

$$\Lambda^{\beta,0}(G) \to \begin{cases} 1 - \frac{1}{n}, & \text{as } \beta \to 0, \\ 1 - \frac{1}{n}\mathscr{H}(H^+), & \text{as } \beta \to 1, \end{cases}$$

where H^+ is the graph obtained by adding a loop to each node in G^+, that is, the adjacency matrix of H^+ is given by $\mathbf{I}_n + \mathbf{A}^+$, and

$$\mathscr{H}(H^+) \equiv \frac{1}{\frac{1}{n}\sum_{i=1}^n \frac{1}{d_i(H^+)}} = \frac{1}{\frac{1}{n}\sum_{i=1}^n \frac{1}{d_i^+ + 1}}$$

is the harmonic mean of the degrees in H^+. The maximum degree of H^+ is bounded by n ($n-1$ in G^+), the harmonic mean $\mathscr{H}(H^+)$ is bounded by n, and it follows from $\frac{\partial \Lambda^{n,\beta}}{\partial \beta} \leq 0$ that

$$0 \leq 1 - \frac{\beta}{n}\mathscr{H}(H^+) \leq \Lambda^{\beta,0}(G) \leq 1 - \frac{1}{n} \leq 1.$$

In the case of $\beta = 1$ (perfect substitutes), $\Lambda^{1,0}(G) = 1 - \frac{1}{n}\mathscr{H}(H^+)$ is an inverse measure of the network density. In a complete graph K_n, we obtain $\Lambda^{1,0}(K_n) = 0$, while in an empty graph $\overline{K}_n$, we get $\Lambda^{1,0}(\overline{K}_n) = 1$. It follows that in the case of $\gamma = 0$ and perfect substitutes when $\beta = 1$, aggregate operational performance vanishes in the complete network, and is highest in the empty network, i.e. the standard Tullock contest success game without effort spillovers.

Further, requiring that $\gamma < 1/d^-_{\max}$ it follows that

$$\Gamma_i^{\beta,\gamma} = \frac{1}{1+\beta d_i^+ - \gamma d_i^-} \geq \Gamma_i^{0,\gamma} = \frac{1}{1-\gamma d_i^-} > 0,$$

and the local hostility levels are non-negative for all $i = 1, \ldots, n$. It then follows that

$$1 - \Lambda^{0,\gamma}(G) = \frac{1}{\sum_{i=1}^n \Gamma_i^{0,\gamma}} \geq 0,$$

which is equivalent to $\Lambda^{0,\gamma}(G) \leq 1$. Hence, we have that $0 \leq \Lambda^{\beta,0}(G) \leq \Lambda^{\beta,\gamma}(G) \leq \Lambda^{0,\gamma}(G) \leq 1$. ∎

We next provide a complete characterization for the key player strategy. The following proposition characterizes the key player in terms of his position in the network.

Proposition 2. *Let $G\backslash\{i\}$ be the network obtained from G by removing agent i and assume that the conditions in Proposition 1 hold. Then the key player $i^* \in \overline{\mathcal{N}} \equiv \{1, \ldots, n\} \cup \emptyset$ is given by*

$$i^* = \arg\max_{i \in \overline{\mathcal{N}}} \left\{ \mathrm{RD}^{\beta,\gamma}(G) - \mathrm{RD}^{\beta,\gamma}(G\backslash\{i\}) \right\},$$

where

$$\begin{aligned}
&\mathrm{RD}^{\beta,\gamma}(G) - \mathrm{RD}^{\beta,\gamma}(G\backslash\{i\}) \\
&= V\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G)) \left\{ \sum_{j=i}^n c_j^{\beta,\gamma}(G) + \sum_{j\neq i} \frac{h_i^{\beta,\gamma}(G)(1-(1-\Lambda^{\beta,\gamma}(G))h_i^{\beta,\gamma}(G))}{1-\Lambda^{\beta,\gamma}(G)} \right. \\
&\times \sum_{k=1}^n \left[\left(m_{jk}^{\beta,\gamma}(G) - \frac{m_{ij}^{\beta,\gamma}(G) m_{ik}^{\beta,\gamma}(G)}{m_{ii}^{\beta,\gamma}(G)} \right) \Gamma_k^{\beta,\gamma}(G) \left(\frac{1+\beta d_k^+ - \gamma d_k^-}{1+\beta(d_k^+ - 1) - \gamma d_k^-} \mathbb{1}_{\{k \in \mathcal{N}_i^+\}} \right.\right. \\
&\left.\left.\left. + \frac{1+\beta d_k^+ - \gamma d_k^-}{1+\beta d_k^+ - \gamma(d_k^- - 1)} \mathbb{1}_{\{k \in \mathcal{N}_i^-\}} + \mathbb{1}_{\{k \notin (\mathcal{N}_i^+ \cup \mathcal{N}_i^-)\}} \right) \right] \right\},
\end{aligned} \tag{41}$$

and we have defined by

$$h_i^{\beta,\gamma}(G) \equiv \left(1 - \beta \sum_{j \in \mathcal{N}_i^+} \frac{\Gamma_j^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))}{1+\beta(d_j^+ - 1) - \gamma d_j^-} - \gamma \sum_{j \in \mathcal{N}_i^-} \frac{\Gamma_j^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))}{1+\beta d_j^+ - \gamma(d_j^- - 1)} \right)^{-1},$$

with $m_{ij}^{\beta,\gamma}(G)$ being the ij-th element of the matrix $\mathbf{M}^{\beta,\gamma}(G) = (\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-)^{-1}$.

Proof of Proposition 2. Let $G\backslash\{i\}$ be the network obtained from G by removing agent i. Then the key player $i^* \in \overline{\mathcal{N}} = \{1, \ldots, n\} \cup \emptyset$ is given by

$$i^* = \arg\max_{i \in \overline{\mathcal{N}}} \left\{ \mathrm{RD}^{\beta,\gamma}(G) - \mathrm{RD}^{\beta,\gamma}(G\backslash\{i\}) \right\}.$$

We can write the change in rent dissipation due to the removal of agent i as follows

$$\mathrm{RD}^{\beta,\gamma}(G)-\mathrm{RD}^{\beta,\gamma}(G\backslash\{i\}) = V\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G)) \times\left(\sum_{j=1}^{n} c_j^{\beta,\gamma}(G) - \frac{\Lambda^{\beta,\gamma}(G\backslash\{i\})(1-\Lambda^{\beta,\gamma}(G\backslash\{i\}))}{\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))}\sum_{j=1}^{n} c_j^{\beta,\gamma}(G\backslash\{i\})\right).$$

With $\sum_{j=1}^{n} c_j^{\beta,\gamma}(G) = \sum_{j=1}^{n} m_{jk}^{\beta,\gamma}(G)\Gamma_k^{\beta,\gamma}(G)$, where $m_{ij}^{\beta,\gamma}(G)$ is the ij-th element of the matrix $\mathbf{M}^{\beta,\gamma}(G)=(\mathbf{I}_n+\beta\mathbf{A}^+-\gamma\mathbf{A}^-)^{-1}$, we can write this as

$$\mathrm{RD}^{\beta,\gamma}(G)-\mathrm{RD}^{\beta,\gamma}(G\backslash\{i\}) = V\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))\Bigg[c_i^{\beta,\gamma}(G)+\sum_{j\neq i}\sum_k\Bigg(m_{jk}^{\beta,\gamma}(G)\Gamma_k^{\beta,\gamma}(G) - \frac{\Lambda^{\beta,\gamma}(G\backslash\{i\})(1-\Lambda^{\beta,\gamma}(G\backslash\{i\}))}{\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))} m_{jk}^{\beta,\gamma}(G\backslash\{i\})\Gamma_k^{\beta,\gamma}(G\backslash\{i\})\Bigg)\Bigg].$$

Using the fact that (cf. Lemma 1 in Ballester, Calvo-Armengol and Zenou 2006)

$$m_{jk}^{\beta,\gamma}(G\backslash\{i\}) = m_{jk}^{\beta,\gamma}(G) - \frac{m_{ij}^{\beta,\gamma}(G)m_{ik}^{\beta,\gamma}(G)}{m_{ii}^{\beta,\gamma}(G)},$$

and denoting by

$$\begin{aligned} h_i^{\beta,\gamma}(G) &\equiv \frac{\Lambda^{\beta,\gamma}(G\backslash\{i\})(1-\Lambda^{\beta,\gamma}(G\backslash\{i\}))}{\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))} \\ &= \left(1-\beta\sum_{j\in\mathcal{N}_i^+}\frac{\Gamma_j^{\beta,\gamma}(1-\Lambda^{\beta,\gamma}(G))}{1+\beta(d_j^+-1)-\gamma d_j^-} - \gamma\sum_{j\in\mathcal{N}_i^-}\frac{\Gamma_j^{\beta,\gamma}(1-\Lambda^{\beta,\gamma}(G))}{1+\beta d_j^+-\gamma(d_j^--1)}\right)^{-1}, \end{aligned}$$

one can show that

$$\begin{aligned} \mathrm{RD}^{\beta,\gamma}(G)-\mathrm{RD}^{\beta,\gamma}(G\backslash\{i\}) = V\Lambda^{\beta,\gamma}(G)(1-\Lambda^{\beta,\gamma}(G))\Bigg\{\sum_{j=1}^{n} c_j^{\beta,\gamma}(G)+\sum_{j\neq i}\frac{h_i^{\beta,\gamma}(G)(1-(1-\Lambda^{\beta,\gamma}(G)))h_i^{\beta,\gamma}(G)}{1-\Lambda^{\beta,\gamma}(G)} \\ \times\sum_{k=1}^{n}\Bigg[\left(m_{jk}^{\beta,\gamma}(G)-\frac{m_{ij}^{\beta,\gamma}(G)m_{ik}^{\beta,\gamma}(G)}{m_{ii}^{\beta,\gamma}(G)}\right)\Gamma_k^{\beta,\gamma}(G)\Bigg(\frac{1+\beta d_k^+-\gamma d_k^-}{1+\beta(d_k^+-1)-\gamma d_k^-}\mathbb{1}_{\{k\in\mathcal{N}_i^+\}} \\ +\frac{1+\beta d_k^+-\gamma d_k^-}{1+\beta d_k^+-\gamma(d_k^--1)}\mathbb{1}_{\{k\in\mathcal{N}_i^-\}}+\mathbb{1}_{\{k\notin(\mathcal{N}_i^+\cup\mathcal{N}_i^-)\}}\Bigg)\Bigg]\Bigg\}, \end{aligned} \tag{42}$$

where $\mathbb{1}_{\{k\in\mathcal{N}_i^+\}}$ and $\mathbb{1}_{\{k\in\mathcal{N}_i^-\}}$ are indicator variables taking the value of one if, respectively, $k\in\mathcal{N}_i^+$and $k\in\mathcal{N}_i^-$, and zero otherwise. Then, the key player can be computed explicitly as

$$\begin{aligned} i^* = \arg\max_{i\in\mathcal{N}}\Bigg\{\sum_{j=1}^{n} c_j^{\beta,\gamma}(G)+\sum_{j\neq i}\frac{h_i^{\beta,\gamma}(G)\left(1-\left(1-\Lambda^{\beta,\gamma}(G)\right)h_i^{\beta,\gamma}(G)\right)}{1-\Lambda^{\beta,\gamma}(G)} \\ \times\sum_{k=1}^{n}\Bigg[\left(m_{jk}^{\beta,\gamma}(G)-\frac{m_{ij}^{\beta,\gamma}(G)m_{ik}^{\beta,\gamma}(G)}{m_{ii}^{\beta,\gamma}(G)}\right)\Gamma_k^{\beta,\gamma}(G)\Bigg(\frac{1+\beta d_k^+-\gamma d_k^-}{1+\beta\left(d_k^+-1\right)-\gamma d_k^-}\mathbb{1}_{\{k\in\mathcal{N}_i^+\}} \\ +\frac{1+\beta d_k^+-\gamma d_k^-}{1+\beta d_k^+-\gamma\left(d_k^--1\right)}\mathbb{1}_{\{k\in\mathcal{N}_i^-\}}+\mathbb{1}_{\{k\notin(\mathcal{N}_i^+\cup\mathcal{N}_i^-)\}}\Bigg)\Bigg]\Bigg\}. \end{aligned} \tag{43}$$

∎

Remark 2. *The key player identified in Proposition 2 differs from the one introduced in Ballester, Calvo-Armengol and Zenou (2006). In the latter, the key player is defined as* $i^* = \arg\max_{i\in\mathcal{N}} b_{\mathbf{u},i}(G,\alpha)/W_i(G,\alpha)$ *with* $b_{\mathbf{u},i}(G,\alpha)$ *being the Bonacich centrality of agent* i *in* G *and* $W_i(G,\alpha)$ *being the generating function of the number of closed walks that start and terminate at node* i.[47] *Compared with our key player formula in equation (41) we find that it is more involved. This is not surprising as the contest success function makes our payoff function generically non-linear.*

In the following we provide a complete equilibrium characterization of the extension we have introduced in Section 2.6. First, let assume that the fighting strength φ_i of agent i depends on an idiosyncratic shifter $\tilde{\varphi}_i$ as in equation (13). Then the following proposition characterizes the local Nash equilibrium.

Proposition 3. *Assume that* $\beta+\gamma < 1/\max\{\lambda_{\max}(G^+), d^-_{\max}\}$, *where* $\lambda_{\max}(\mathbf{A}^{\pm})$ *denotes the largest eigenvalue associated with the matrix* $\mathbf{A}^{\pm}$. *Let* $\Gamma_i^{\beta,\gamma}(G)$ *and* $\Lambda^{\beta,\gamma}(G)$ *be defined as in equation (6), and let*

$$\mathbf{c}_{\boldsymbol{\mu}}^{\beta,\gamma}(G) \equiv \left(\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-\right)^{-1}\boldsymbol{\mu} \tag{44}$$

be a centrality vector, whose generic element $c_{\boldsymbol{\mu},i}^{\beta,\gamma}(G)$ *describes the centrality of agent* i *in the network for some vector* $\boldsymbol{\mu}\in\mathbb{R}^n$. *Then there exists a unique local Nash equilibrium of the* $n-$*player simultaneous move game with payoffs given by equation (1), agents' OPs in equation (2) and strategy space* $\mathcal{S} = [\underline{x}_1,\infty)\times[\underline{x}_2,\infty)\times\ldots\times[\underline{x}_n,\infty)\subset\mathbb{R}^n$, *where the equilibrium effort levels are given by*

$$x_i^*(G) = V\Lambda^{\beta,\gamma}(G)\left(1-\Lambda^{\beta,\gamma}(G)\right)c^{\beta,\gamma}_{\mathbf{\Gamma}^{\beta,\gamma}(G),i}(G) - c^{\beta,\gamma}_{\tilde{\boldsymbol{\varphi}},i}(G), \tag{45}$$

for all $i=1,\ldots,n$. *Moreover, the equilibrium OPs are given by equation (26), and the equilibrium payoffs are given by*

$$\pi_i^*(G) \equiv \pi_i(\mathbf{x}^*,G) = V(1-\Lambda^{\beta,\gamma}(G))\left(\Gamma_i^{\beta,\gamma}(G) - \Lambda^{\beta,\gamma}(G)\,c^{\beta,\gamma}_{\mathbf{\Gamma}^{\beta,\gamma}(G),i}(G)\right) + c^{\beta,\gamma}_{\tilde{\boldsymbol{\varphi}},i}(G). \tag{46}$$

Proof of Proposition 3. In order to compute the equilibrium we start with deriving the necessary first order conditions (FOCs). With equation (13) we can write the payoff of agent i as follows

$$\begin{aligned}\pi_i(G,\mathbf{x}) &= V\frac{\varphi_i}{\sum_{j=1}^n\varphi_j} - x_i\\ &= V\frac{x_i + \beta\sum_{j=1}^n a^+_{ij}x_j - \gamma\sum_{j=1}^n a^-_{ij}x_j + \tilde{\varphi}_i}{\sum_{j=1}^n\left(x_j + \beta\sum_{k=1}^n a^+_{jk}x_k - \gamma\sum_{k=1}^n a^-_{jk}x_k + \tilde{\varphi}_j\right)} - x_i.\end{aligned} \tag{47}$$

The partial derivatives are given by

$$\begin{aligned}\frac{\partial\pi_i}{\partial x_i} &= V\frac{\frac{\partial\varphi_i}{\partial x_i}\sum_{j=1}^n\varphi_j - \varphi_i\sum_{j=1}^n\frac{\partial\varphi_j}{\partial x_i}}{\left(\sum_{j=1}^n\varphi_j\right)^2} - 1\\ &= V\frac{\sum_{j=1}^n\varphi_j - \varphi_i(1+\beta d_i^+ - \gamma d_i^-)}{\left(\sum_{j=1}^n\varphi_j\right)^2} - 1,\end{aligned} \tag{48}$$

[47] See also equation (61) in the Online Appendix D.

where we have used the fact that $\frac{\partial \varphi_j}{\partial x_i} = \delta_{ij} + \beta a_{ij}^+ - \gamma a_{ij}^-$ and consequently $\sum_{j=1}^n \frac{\partial \varphi_j}{\partial x_i} = 1 + \beta d_i^+ - \gamma d_i^-$. The first order conditions are then given by $\frac{\partial \pi_i}{\partial x_i} = 0$. From the partial derivative in equation (48) the FOC can be written as follows

$$\frac{\partial \pi_i}{\partial x_i} = V \frac{\sum_{j=1}^n \varphi_j - \varphi_i (1 + \beta d_i^+ - \gamma d_i^-)}{\left(\sum_{j=1}^n \varphi_j\right)^2} - 1 = 0,$$

from which we get

$$\varphi_i = \frac{1}{1 + \beta d_i^+ - \gamma d_i^-} \left(1 - \frac{1}{V} \sum_{j=1}^n \varphi_j\right) \sum_{j=1}^n \varphi_j.$$

Summation over i gives

$$\sum_{i=1}^n \varphi_i = V \left(1 - \frac{1}{\sum_{i=1}^n \frac{1}{1+\beta d_i^+ - \gamma d_i^-}}\right).$$

With $\Gamma_i^{\beta,\gamma}(G)$ and $\Lambda^{\beta,\gamma}(G)$ as in equation (6) we can write the aggregate operational performance as

$$\sum_{i=1}^n \varphi_i = V \Lambda^{\beta,\gamma}(G),$$

which is equivalent to equation (26). The individual operational performance can be written as

$$\varphi_i(G, \mathbf{x}) = V \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G)) \Gamma_i^{\beta,\gamma}(G), \tag{49}$$

which is equivalent to equation (26). We then get

$$\varphi_i(G, \mathbf{x}) = x_i + \beta \sum_{j=1}^n a_{ij}^+ x_j - \gamma \sum_{j=1}^n a_{ij}^- x_j + \tilde{\varphi}_i = V \Lambda^{n,\beta}(G)(1 - \Lambda^{n,\beta}(G)) \Gamma_i^{\beta,\gamma}(G). \tag{50}$$

We can write

$$x_i + \beta \sum_{j=1}^n a_{ij}^+ x_j - \gamma \sum_{j=1}^n a_{ij}^- x_j = \varphi_i - \tilde{\varphi}_i = V \Lambda^{n,\beta}(G)(1 - \Lambda^{n,\beta}(G)) \Gamma_i^{\beta,\gamma}(G) - \tilde{\varphi}_i. \tag{51}$$

Denoting by $\mathbf{\Gamma}^{\beta,\gamma}(G) \equiv (\Gamma_1^{\beta,\gamma}(G), \ldots, \Gamma_n^{\beta,\gamma}(G))^\top$, we can write this in vector-matrix form as

$$(\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)\mathbf{x} = V \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G)) \mathbf{\Gamma}^{\beta,\gamma}(G) - \tilde{\boldsymbol{\varphi}}.$$

When the matrix $\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-$ is invertible (see the proof of Proposition 1 for the conditions that guarantee invertibility), we obtain a unique solution given by

$$\mathbf{x} = V \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))(\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)^{-1} \mathbf{\Gamma}^{\beta,\gamma}(G) - (\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)^{-1} \tilde{\boldsymbol{\varphi}}. \tag{52}$$

With the definition of the centrality in equation (44) we then can write equation (52) in the form of equation (45) in the proposition. Moreover, using the fact that equilibrium payoffs are given by $\pi_i(\mathbf{x}^*, G) = V \frac{\varphi_i^*(G)}{\sum_{j=1}^n \varphi_j^*(G)} - x_i^*$ we obtain equation (46) in the proposition.

Further, the second order partial derivatives, $\frac{\partial^2 \pi_i}{\partial x_i^2}$, are the same as in the proof of Proposition 1. In particular, it follows that $\frac{\partial^2 \pi_i}{\partial x_i^2} < 0$ if $\Lambda^{\beta,\gamma}(G) > 0$ and $1 + \beta d_i^+ - \gamma d_i^- > 0$. The last inequality requires that the local hostility level of agent i is positive, i.e. $\Gamma_i^{\beta,\gamma}(G) > 0$. It then follows that the equilibrium payoff function is locally concave at the efforts levels in equation (52).

As in the proof of Proposition 1 in the remainder of the proof we show that the equilibrium payoff function is globally concave, by bounding the effort levels (from below). This implies that the marginal payoff of an agent from deviating from the Nash equilibrium strategy is always negative. Observe that

$$\begin{aligned}\pi_i(\mathbf{x}_{-i}^*, x_i, G) &= V \frac{\varphi_i(\mathbf{x}_{-i}^*, x, G)}{\sum_{j=1}^n \varphi_j(\mathbf{x}_{-i}^*, x, G)} - x_i \\ &= V \frac{\varphi_i(\mathbf{x}^*, G) - (x_i^* - x_i)}{\sum_{j=1}^n \varphi_j(\mathbf{x}^*, G) - (1 + \beta d_i^+ - \gamma d_i^-)(x_i^* - x_i)} - x_i \\ &= V \frac{A_i^{\beta,\gamma}(G) + \Gamma_i^{\beta,\gamma}(G) x_i}{B_i^{\beta,\gamma}(G) + x_i} - x_i, \end{aligned} \tag{53}$$

where we have denoted by

$$\begin{aligned} A_i^{\beta,\gamma}(G) &\equiv \Gamma_i^{\beta,\gamma}(G)(\varphi_i^* - x_i^*) = V\Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))\Gamma_i^{\beta,\gamma}(G)\left(\Gamma_i^{\beta,\gamma}(G) - c_i^{\beta,\gamma}(G)\right) + \Gamma_i^{\beta,\gamma}(G)\tilde{c}_i^{\beta,\gamma}(G), \\ B_i^{\beta,\gamma}(G) &\equiv \Gamma_i^{\beta,\gamma}(G)\sum_{j=1}^n \varphi_j^* - x_i^* = V\Lambda^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) - x_i^* \\ &= V\Lambda^{\beta,\gamma}(G)\left(\Gamma_i^{\beta,\gamma}(G) - (1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G)\right) + \tilde{c}_i^{\beta,\gamma}(G), \end{aligned}$$

the centrality $c_i^{\beta,\gamma}(G)$ has been defined in equation (7) and we have denoted by $\tilde{\mathbf{c}}^{\beta,\gamma}(G) \equiv (\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-)^{-1}\tilde{\boldsymbol{\varphi}}$. Assume first that $B_i^{\beta,\gamma}(G) \geq 0$. For $x_i > -B_i^{\beta,\gamma}(G)$ the function $\pi_i(\mathbf{x}_{-i}^*, x, G)$ is concave in x_i with a maximum at x_i^*, and decreasing for values of $x_i > x_i^*$ with an asymptote given by $-x_i$.[48] In particular, if x_i is larger than the discontinuity at $-B_i^{\beta,\gamma}(G)$ of $\pi_i(\mathbf{x}_{-i}^*, x, G)$ then we have that $\pi_i(\mathbf{x}_{-i}^*, x_i, G) < \pi_i(\mathbf{x}^*, G)$. In particular, we have that $\pi_i(\mathbf{x}_{-i}^*, x_i, G) < \pi_i(\mathbf{x}^*, G)$ iff

$$\begin{aligned} V\frac{A_i^{\beta,\gamma}(G) + \Gamma_i^{\beta,\gamma}(G)x_i}{B_i^{\beta,\gamma}(G) + x_i} - x_i < \pi_i^* &= V\frac{\varphi_i^*}{\sum_{j=1}^n \varphi_j^*(G)} - x_i^* \\ &= V(1 - \Lambda^{\beta,\gamma}(G))\Gamma_i^{\beta,\gamma}(G) - x_i^* \\ &= V(1 - \Lambda^{\beta,\gamma}(G))\left(\Gamma_i^{\beta,\gamma}(G) - \Lambda^{\beta,\gamma}(G)c_i^{\beta,\gamma}(G)\right) + \tilde{c}_i^{\beta,\gamma}(G), \end{aligned} \tag{54}$$

where we have denoted by $\pi_i^* \equiv \pi_i(\mathbf{x}^*, G)$. Next, observe that from the FOC

$$\frac{\partial \pi_i(\mathbf{x}_{-i}^*, x_i, G)}{\partial x_i} = -\frac{A_i^{\beta,\gamma}(G) - B_i^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) + (B_i^{\beta,\gamma}(G) + x_i)^2}{(B_i^{\beta,\gamma}(G) + x_i)^2} = 0, \tag{55}$$

it follows that[49]

$$x_i = -B_i^{\beta,\gamma}(G) + \sqrt{B_i^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) - A_i^{\beta,\gamma}(G)} = x_i^*,$$

[48] See footnote 45.

[49] W.l.o.g. assume that $V = 1$, Then using the fact that $A_i^{\beta,\gamma}(G) = \Gamma_i^{\beta,\gamma}(G)\Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))(\Gamma_i^{\beta,\gamma}(G) - c_i^{\beta,\gamma}(G)) + \Gamma_i^{\beta,\gamma}(G)\tilde{c}_i^{\beta,\gamma}(G)$, $B_i^{\beta,\gamma}(G) = \Lambda^{\beta,\gamma}(G)(\Gamma_i^{\beta,\gamma}(G) - (1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G)) + \tilde{c}_i^{\beta,\gamma}(G)$ and $x_i^* = \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G) - \tilde{c}_i^{\beta,\gamma}(G)$, we find that $-B_i^{\beta,\gamma}(G) + \sqrt{B_i^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G) - A_i^{\beta,\gamma}(G)} = \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G) - \tilde{c}_i^{\beta,\gamma}(G) = x_i^*$.

and inserting into $\pi_i(\mathbf{x}^*_{-i}, x_i, G)$ yields π_i^*. We then define $\underline{x}_i \equiv -B_i^{\beta,\gamma}(G)$. When $B_i^{\beta,\gamma}(G) \geq 0$ it then follows that $\underline{x}_i < 0$. Note that if $B_i^{\beta,\gamma}(G) > 0$ then the sign of $A_i^{\beta,\gamma}(G)$ does not qualitatively change the functional form of $\pi_i(\mathbf{x}^*_{-i}, x, G)$ (more precisely, $\pi_i(\mathbf{x}^*_{-i}, x, G)$ is a concave function for $x_i \geq -B_i^{\beta,\gamma}(G)$ with a global maximum over its domain at x_i^*; see also footnote 45). In contrast, if $B_i^{\beta,\gamma}(G) < 0$ then we must have that $A_i^{\beta,\gamma}(G)$ must be negative as well. More precisely, if $B_i^{\beta,\gamma}(G) < 0$ then we must have that $A_i^{\beta,\gamma}(G) < B_i^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G)$. Otherwise we would have that $\frac{\partial \pi_i(\mathbf{x}^*_{-i}, x_i, G)}{\partial x_i} < 0$ (see equation (55)), violating the FOC. However, under this condition, the functional form of $\pi_i(\mathbf{x}^*_{-i}, x, G)$ remains qualitatively unchanged. Importantly, for all the above cases we find that $\pi_i(\mathbf{x}^*_{-i}, x_i, G) - \pi_i(\mathbf{x}^*, G) < 0$ if $x_i \in [\underline{x}_i, \infty)$, and we obtain a local Nash equilibrium (see Definition 2). This completes the proof. ∎

Remark 3. *Similar to Remark 1 it is possible to compute a common lower bound, $\underline{x}$, on the effort levels in Proposition 3 such that $\pi_i(\mathbf{x}^*_{-i}, x_i, G) - \pi_i(\mathbf{x}^*, G) < 0$ holds for all $i = 1, \ldots, n$ if $\mathbf{x} \in [\underline{x}, \infty)^n$. We also require that $\underline{x} < x_i^*$ for all $i = 1, \ldots, n$. These conditions then imply a symmetric local Nash equilibrium (see Definition 2). We denote this common lower bound as $\underline{x} \equiv \max_{i=1,\ldots,n} \underline{x}_i$, and we require that $\forall i = 1, \ldots, n$*

$$\underline{x} \leq x_i^* = V\Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G) - \tilde{c}_i^{\beta,\gamma}(G). \tag{56}$$

Observe that equation (56) is equivalent to

$$\max_{i=1,\ldots,n} \left\{-B_i^{\beta,\gamma}(G)\right\} = \max_{i=1,\ldots,n} \left\{ x_i^* - \Gamma_i^{\beta,\gamma}(G) \sum_{j=1}^{n} \varphi_j^* \right\} \leq \min_{i=1,\ldots,n} x_i^*,$$

which is

$$\max_{i=1,\ldots,n} \left\{x_i^* - V\Lambda^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G)\right\} \leq \min_{i=1,\ldots,n} x_i^*,$$

or equivalently

$$\max_{i=1,\ldots,n} \left\{V\Lambda^{\beta,\gamma}(G)\left((1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G) - \Gamma_i^{\beta,\gamma}(G)\right) - \tilde{c}_i^{\beta,\gamma}(G)\right\}$$
$$\leq \min_{i=1,\ldots,n} \left\{V\Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G))c_i^{\beta,\gamma}(G) - \tilde{c}_i^{\beta,\gamma}(G)\right\}. \tag{57}$$

Online Appendix C Tables for Key Player Analysis and Pacification Policies

In this appendix we provide additional tables complementing the discussion in Sections 5.2 and 5.3.

Appendix Table I: Key player ranking for the first 35 actors in the Democratic Republic of the Congo (DRC) excluding government groups.[a] The total number of groups is $n = 85$. The total number of alliances is $m^{+}(G^{b}) = 95$, the total number of conflicts is $m^{-}(G^{b}) = 111$ and we have that $\sum_j \varphi_j^*(G^b) = 0.9891$. The Pearson correlation of fighting effort in the benchmark with the reduction in the rent dissipation is 0.76.

Actor	$\Delta\mathrm{RD}_k^{\hat{\beta},\hat{\gamma}}$ (%)[b]	$\varphi_i^*(G^b)/\sum_j \varphi_j^*(G^b)$ (%)	$x_i^*(G^b)/\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b)$ (%)	Rank $x_i^*(G^b)$	Rank $\Delta\mathrm{RD}_k^{\hat{\beta},\hat{\gamma}}$
FDLR: Dem. Forces for the Lib. of Rwanda	13.79	0.71	6.65	4	1
Mayi-Mayi Milita	9.67	0.69	5.59	6	2
Mil. Forc. of Rwanda (2000-)	9.57	1.91	3.65	10	3
Mil. Forc. of Uganda (1986-)	7.71	0.93	4.34	8	4
Lendu Ethnic Militia (DRC)	4.97	1.24	2.43	13	5
Mil. Forc. of Rwanda (1994-1999)	4.56	1.18	1.69	16	6
RCD: Rally for Congolese Dem.	4.49	0.98	1.75	15	7
Interahamwe Hutu Ethnic Militia	2.36	0.84	1.39	18	8
RCD: Rally for Congolese Dem. (Goma)	2.36	6.90	8.67	3	9
UPC: Union of Congolese Patriots	1.74	1.58	2.16	14	10
Mil. Forc. of Zimbabwe (1980-)	1.72	0.67	0.73	24	11
Hutu Rebels	1.46	0.99	0.52	27	12
Mil. Forc. of South Africa (1994-1999)	0.80	0.91	0.28	35	13
FAA/MPLA: Mil. Forc. of Angola (1975-)	0.75	0.76	0.36	31	14
Mayi Mayi Militia (PARECO)	0.72	0.68	0.37	29	15
Mil. Forc. of Namibia (1990-2005)	0.62	0.76	0.31	32	16
PUSIC: Party Unity & Safe. of Congo's Integrity	0.54	1.33	0.26	36	17
Former Mil. Forc. of Rwanda (1973-1994)	0.45	0.79	0.24	37	18
Mil. Forc. of Sudan (1993-)	0.41	0.95	0.24	38	19
Mil. Forc. of Burundi (1996-2005)	0.27	1.20	0.29	33	20
Mayi-Mayi Militia (Yakutumba)	0.25	0.70	0.15	42	21
PPRD: People's Party for Reconstr. & Dem.	0.13	1.09	0.13	43	22
ADFL: All. of Dem. Forces for Lib. (Congo-Zaire) (1996-1997)	0.11	1.33	0.06	55	23
Munzaya Ethnic Militia (DRC)	0.11	1.09	0.11	45	24
Mil. Forc. of Zambia (1991-2002)	0.10	1.20	0.05	62	25
Mayi Mayi Milita (Cmdt La Fontaine)	0.10	0.82	0.06	56	26
Mayi-Mayi Militia (Cmdt Jackson)	0.10	0.82	0.06	57	27
Wageregere Ethnic Militia	0.10	1.20	0.05	61	28
Mil. Forc. of Chad (1990-)	0.08	0.82	0.05	63	29
RCD: Rally for Congolese Dem. (Nat.)	0.06	0.70	0.03	72	30
Ngiti Ethnic Militia	0.06	0.96	0.03	70	31
Bomboma Ethnic Militia	0.06	1.20	0.08	50	32
RUD: Gathering for Unity & Dem.	0.06	1.20	0.05	60	33
Mayi-Mayi Militia (Kifuafua)	0.05	0.77	0.03	73	34
Alur Ethnic Militia (Uganda)	0.05	0.96	0.03	71	35

[a] We have used the parameter estimates $\hat{\beta} = 0.1407$ and $\hat{\gamma} = 0.0903$ while setting $V = 1$.

[b] The relative change in the rent dissipation is computed as $\Delta\mathrm{RD}_k^{\hat{\beta},\hat{\gamma}}(\%) = \left(\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b) - \mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b \backslash \{k\})\right)/\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b)$, while the rent dissipation in the original network G^b is $\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b) = \sum_{i=1}^{n} x_i^*(G^b) = 474.4614$ (with $V = 1$). A positive value of $\Delta\mathrm{RD}_k^{\hat{\beta},\hat{\gamma}}$ thus indicates a reduction in fighting, while a negative value indicates an increase in fighting.

(continued) Appendix Table I: Key player ranking for the 36-th to the 80-th actors in the Democratic Republic of the Congo (DRC) excluding government groups.

Actor	$\Delta \mathrm{RD}_k^{\hat{\beta},\hat{\gamma}}$ (%)	$\varphi_i^*(G^b)/\sum_j \varphi_j^*(G^b)$ (%)	$x_i^*(G^b)/\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b)$ (%)	Rank $x_i^*(G^b)$	Rank $\Delta \mathrm{RD}_k^{\hat{\beta},\hat{\gamma}}$
Lendu Ethnic Militia (Uganda)	0.04	1.20	0.06	54	36
FLC: Congolese Lib. Front	0.04	1.49	0.16	40	37
Group of 47	0.03	1.09	0.03	68	38
Banyamulenge Ethnic Militia (DRC)	0.03	1.09	0.03	67	39
Mbingi Community Militia (DRC)	0.03	1.09	0.03	69	40
Mayi Mayi Militia (Mbuayi)	0.03	0.77	0.02	83	41
All. for Dem. Change	0.02	0.96	0.02	82	42
DSP: Division Speciale Presidentelle	0.02	0.96	0.02	81	43
FAP: Popular Self-Defense Forces	0.01	1.09	0.02	74	44
Hutu Refugees (Rwanda)	0.01	1.09	0.02	77	45
Mutiny of LRA: Lord's Resistance Army	0.01	1.09	0.02	75	46
PRA: People's Redemption Army	0.01	1.09	0.02	76	47
Mil. Forc. of Zaire (1965-1997)	0.01	1.09	0.02	78	48
Wangilima Ethnic Militia	0.01	1.09	0.02	79	49
Pygmy Ethnic Group (DRC)	0.01	1.09	0.02	80	50
Faustin Munene Militia (DRC)	0.00	1.09	0.00	84	51
UNAFEC: Union of Nat. Fed. of Congo Party (DRC)	0.00	1.09	0.00	85	52
CNDD-FDD (Ndayikengurukiye faction)	-0.03	1.04	0.03	66	53
RCD: Rally for Congolese Dem. (Masunzu)	-0.04	0.82	0.05	59	54
UPPS: Union for Dem. & Social Progr. Party (DRC)	-0.04	1.33	0.03	65	55
FRF: Federal Republican Forces	-0.08	1.20	0.05	58	56
Haut-Uele Resident Militia	-0.09	1.04	0.06	53	57
Minembwe Dissidents	-0.10	1.20	0.06	52	58
APCLS: All. of Patriots for a Free & Sov. Congo	-0.11	1.04	0.06	51	59
Unnamed Mayi-Mayi Militia (DRC)	-0.14	1.04	0.08	49	60
Lobala (Enyele) Militia	-0.14	1.33	0.10	46	61
FNL: Nat. Forces of Lib.	-0.16	1.14	0.10	47	62
Enyele Ethnic Militia (DRC)	-0.17	1.20	0.11	44	63
UNITA: Nat. Union for the Total Indep. of Angola	-0.27	0.99	0.15	41	64
BDK: Bunda Dia Kongo	-0.29	1.20	0.19	39	65
CNDD-FDD: Nat. Council for the Defence of Dem.	-0.37	1.71	0.71	25	66
ALIR: Army for the Lib. of Rwanda	-0.42	1.33	0.36	30	67
NALU: Nat. Army for the Lib. of Uganda	-0.56	0.99	0.28	34	68
MRC: Rev. Movement of Congo	-0.56	0.91	0.47	28	69
SPLA/M: Sudanese People's Lib. Army/Movement	-0.87	1.04	0.73	23	70
FNI: Nat. & Integr. Front	-1.17	1.04	0.84	22	71
FPJC: Popular Front for Justice in Congo	-1.32	1.33	0.65	26	72
FRPI: Front for Patr. Resist. of Ituri	-1.34	1.14	0.86	21	73
Hema Ethnic Militia (DRC)	-1.78	0.98	0.94	20	74
ADF: Allied Dem. Forces	-2.40	1.08	1.33	19	75
Mutiny of Mil. Forc. of DRC (2003-)	-3.30	1.08	1.64	17	76
MLC: Congolese Lib. Movement	-5.22	1.07	3.08	11	77
RCD: Rally for Congolese Dem. (Kisangani)	-5.46	4.39	5.95	5	78
LRA: Lord's Resistance Army	-7.54	2.96	4.13	9	79
CNDP: Nat. Congress for the Defense of the People	-8.78	1.18	4.35	7	80

Appendix Table II: Key links ranking for the first 35 actors in the Democratic Republic of the Congo (DRC) excluding government groups.[a] The total number of groups is $n = 85$. The total number of alliances is $m^{+}(G^b) = 95$, the total number of conflicts is $m^{-}(G^b) = 111$ and we have that $\sum_j \varphi_j^*(G^b) = 0.9891$. The Pearson correlation of fighting effort in the benchmark with the reduction in the rent dissipation is 0.72.

Actor	$\Delta\mathrm{RD}_{kj}^{\hat{\beta},\hat{\gamma}}$ (%)[b]	$\varphi_i^*(G^b)/\sum_j \varphi_j^*(G^b)$ (%)	$x_i^*(G^b)/\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b)$ (%)	Rank $x_i^*(G^b)$	Rank $\Delta\mathrm{RD}_{kj}^{\hat{\beta},\hat{\gamma}}$
RCD: Rally for Congolese Dem. (Goma)	17.21	6.90	8.67	3	1
FDLR: Dem. Forces for the Lib. of Rwanda	9.68	0.71	6.65	4	2
RCD: Rally for Congolese Dem. (Kisangani)	8.74	4.39	5.95	5	3
LRA: Lord's Resistance Army	4.30	2.96	4.13	9	4
CNDP: Nat. Congress for the Defense of the People	2.53	1.18	4.35	7	5
Lendu Ethnic Militia (DRC)	2.47	1.24	2.43	13	6
Mil. Forc. of Rwanda (2000-)	0.60	1.91	3.65	10	7
Mil. Forc. of Zimbabwe (1980-)	0.29	0.67	0.73	24	8
Mayi-Mayi Militia (Cmdt Jackson)	0.27	0.82	0.06	57	9
Mayi Mayi Milita (Cmdt La Fontaine)	0.27	0.82	0.06	56	10
Mil. Forc. of Uganda (1986-)	0.27	0.93	4.34	8	11
Mayi-Mayi Milita	0.25	0.69	5.59	6	12
Mil. Forc. of Sudan (1993-)	0.25	0.95	0.24	38	13
Mayi Mayi Militia (Mbuayi)	0.24	0.77	0.02	83	14
Mayi-Mayi Militia (Kifuafua)	0.24	0.77	0.03	73	15
Mil. Forc. of Chad (1990-)	0.24	0.82	0.05	63	16
Mayi-Mayi Militia (Yakutumba)	0.24	0.70	0.15	42	17
Former Mil. Forc. of Rwanda (1973-1994)	0.23	0.79	0.24	37	18
Mayi Mayi Militia (PARECO)	0.23	0.68	0.37	29	19
FAA/MPLA: Mil. Forc. of Angola (1975-)	0.22	0.76	0.36	31	20
Mil. Forc. of Namibia (1990-2005)	0.22	0.76	0.31	32	21
Interahamwe Hutu Ethnic Militia	0.20	0.84	1.39	18	22
MLC: Congolese Lib. Movement	-1.98	1.07	3.08	11	23
Mil. Forc. of Rwanda (1994-1999)	-3.08	1.18	1.69	16	24
UPC: Union of Congolese Patriots	-3.27	1.58	2.16	14	25
RCD: Rally for Congolese Dem.	-3.43	0.98	1.75	15	26
Hutu Rebels	-5.28	0.99	0.52	27	27
Mil. Forc. of South Africa (1994-1999)	-5.68	0.91	0.28	35	28
PUSIC: Party Unity & Safe. of Congo's Integrity	-7.20	1.33	0.26	36	29
Mil. Forc. of Zambia (1991-2002)	-7.56	1.20	0.05	62	30
Wageregere Ethnic Militia	-7.75	1.20	0.05	61	31
RCD: Rally for Congolese Dem. (Nat.)	-7.76	0.70	0.03	72	32
Alur Ethnic Militia (Uganda)	-8.01	0.96	0.03	71	33
All. for Dem. Change	-8.07	0.96	0.02	82	34
Ngiti Ethnic Militia	-8.14	0.96	0.03	70	35

[a] We have used the parameter estimates $\hat{\beta} = 0.1407$ and $\hat{\gamma} = 0.0903$ while setting $V = 1$.

[b] The relative change in the rent dissipation is computed as $\Delta\mathrm{RD}_{kj}^{\hat{\beta},\hat{\gamma}}(\%) = \left(\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b) - \mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b\backslash\{kj\})\right)/\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b)$, while the rent dissipation in the original network G^b is $\mathrm{RD}^{\hat{\beta},\hat{\gamma}}(G^b) = \sum_{i=1}^n x_i^*(G^b) = 474.4614$ (with $V = 1$). A positive value of $\Delta\mathrm{RD}_{kj}^{\hat{\beta},\hat{\gamma}}$ thus indicates a reduction in fighting, while a negative value indicates an increase in fighting.

(continued) Appendix Table II: Key links ranking for the 36-th to the 80-th actors in the Democratic Republic of the Congo (DRC) excluding government groups.

Actor	$\Delta \mathrm{RD}_{kj}^{\hat\beta,\hat\gamma}$ (%)	$\varphi_i^*(G^b)/\sum_j \varphi_j^*(G^b)$ (%)	$x_i^*(G^b)/\mathrm{RD}^{\hat\beta,\hat\gamma}(G^b)$ (%)	Rank $x_i^*(G^b)$	Rank $\Delta \mathrm{RD}_{kj}^{\hat\beta,\hat\gamma}$
DSP: Division Speciale Presidentelle	-8.19	0.96	0.02	81	36
Mil. Forc. of Burundi (1996-2005)	-8.20	1.20	0.29	33	37
ADFL: All. of Dem. Forces for Lib. (Congo-Zaire) (1996-1997)	-8.23	1.33	0.06	55	38
PPRD: People's Party for Reconstr. & Dem.	-8.49	1.09	0.13	43	39
RUD: Gathering for Unity & Dem.	-8.54	1.20	0.05	60	40
Munzaya Ethnic Militia (DRC)	-8.54	1.09	0.11	45	41
CNDD-FDD: Nat. Council for the Defence of Dem.	-8.60	1.71	0.71	25	42
Banyamulenge Ethnic Militia (DRC)	-8.80	1.09	0.03	67	43
Group of 47	-8.80	1.09	0.03	68	44
Mbingi Community Militia (DRC)	-8.80	1.09	0.03	69	45
FAP: Popular Self-Defense Forces	-8.85	1.09	0.02	74	46
PRA: People's Redemption Army	-8.85	1.09	0.02	76	47
Mutiny of LRA: Lord's Resistance Army	-8.85	1.09	0.02	75	48
Hutu Refugees (Rwanda)	-8.85	1.09	0.02	77	49
Mil. Forc. of Zaire (1965-1997)	-8.85	1.09	0.02	78	50
Wangilima Ethnic Militia	-8.85	1.09	0.02	79	51
Pygmy Ethnic Group (DRC)	-8.85	1.09	0.02	80	52
Faustin Munene Militia (DRC)	-8.90	1.09	-0.00	84	53
UNAFEC: Union of Nat. Fed. of Congo Party (DRC)	-8.90	1.09	-0.00	85	54
Bomboma Ethnic Militia	-9.05	1.20	0.08	50	55
FLC: Congolese Lib. Front	-9.18	1.49	0.16	40	56
Lendu Ethnic Militia (Uganda)	-9.28	1.20	0.06	54	57
SPLA/M: Sudanese People's Lib. Army/Movement	-9.84	1.04	0.73	23	58
FNI: Nat. & Integr. Front	-10.15	1.04	0.84	22	59
MRC: Rev. Movement of Congo	-10.34	0.91	0.47	28	60
RCD: Rally for Congolese Dem. (Masunzu)	-10.90	0.82	0.05	59	61
Unnamed Mayi-Mayi Militia (DRC)	-10.97	1.04	0.08	49	62
ALIR: Army for the Lib. of Rwanda	-10.99	1.33	0.36	30	63
FRPI: Front for Patr. Resist. of Ituri	-11.16	1.14	0.86	21	64
FNL: Nat. Forces of Lib.	-11.44	1.14	0.10	47	65
Hema Ethnic Militia (DRC)	-11.52	0.98	0.94	20	66
Mutiny of Mil. Forc. of DRC (2003-)	-11.90	1.08	1.64	17	67
CNDD-FDD (Ndayikengurukiye faction)	-12.13	1.04	0.03	66	68
NALU: Nat. Army for the Lib. of Uganda	-12.72	0.99	0.28	34	69
Haut-Uele Resident Militia	-13.06	1.04	0.06	53	70
UNITA: Nat. Union for the Total Indep. of Angola	-13.48	0.99	0.15	41	71
BDK: Bunda Dia Kongo	-13.86	1.20	0.19	39	72
Enyele Ethnic Militia (DRC)	-14.22	1.20	0.11	44	73
UPPS: Union for Dem. & Social Progr. Party (DRC)	-14.26	1.33	0.03	65	74
Minembwe Dissidents	-14.44	1.20	0.06	52	75
FRF: Federal Republican Forces	-14.51	1.20	0.05	58	76
Lobala (Enyele) Militia	-14.53	1.33	0.10	46	77
APCLS: All. of Patriots for a Free & Sov. Congo	-14.73	1.04	0.06	51	78
FPJC: Popular Front for Justice in Congo	-16.73	1.33	0.65	26	79
ADF: Allied Dem. Forces	-37.28	1.08	1.33	19	80

Online Appendix D Bonacich Centrality

In this section we discuss a network measure capturing the centrality of an agent in the network due to Bonacich (1987). Let $\mathbf{A}$ be the symmetric $n \times n$ adjacency matrix of the network G and $\lambda_{\max}$ its largest real eigenvalue. The matrix $\mathbf{M}(G,\alpha) = (\mathbf{I} - \alpha\mathbf{A})^{-1}$ exists and is non-negative if and only if $\alpha < 1/\lambda_{\max}$.[50] Under this condition it can be written as

$$\mathbf{M}(G,\alpha) = \sum_{k=0}^{\infty} \alpha^k \mathbf{A}^k. \tag{58}$$

The vector of Bonacich centralities is then given by

$$\mathbf{b}_{\mathbf{u}}(G,\alpha) = \mathbf{M}(G,\alpha)\mathbf{u}, \tag{59}$$

where $\mathbf{u} = (1,\dots,1)^\top$ is an n-dimensional vector of ones. We can write the vector of Bonacich centralities as

$$\mathbf{b}_{\mathbf{u}}(G,\alpha) = (\mathbf{I} - \alpha\mathbf{A})^{-1}\mathbf{u} = \sum_{k=0}^{\infty} \alpha^k \mathbf{A}^k \mathbf{u}.$$

For the components $b_{\mathbf{u},i}(G,\alpha)$, $i = 1,\dots,n$, we get

$$b_{\mathbf{u},i}(G,\alpha) = \sum_{k=0}^{\infty} \alpha^k (\mathbf{A}^k \cdot \mathbf{u})_i = \sum_{k=0}^{\infty} \alpha^k \sum_{j=1}^{n} a_{ij}^{[k]}, \tag{60}$$

where $a_{ij}^{[k]}$ is the ij-th element of $\mathbf{A}^k$. Because $N_{k,i}(G) \equiv \sum_{j=1}^{n} a_{ij}^{[k]}$ counts the number of all walks of length k in G starting from i, the Bonacich centrality of agent i, $b_{\mathbf{u},i}(G,\alpha)$, is thus equivalent to the number of all walks in G starting from i, where the walks of length k are weighted by a geometrically decaying factor α^k.

Further, the sum of the Bonacich centralities, $\sum_{i=1}^{n} b_{\mathbf{u},i}(G,\alpha) = \mathbf{u}^\top \mathbf{b}_{\mathbf{u}}(G,\alpha) = \mathbf{u}^\top \mathbf{M}(G,\alpha)\mathbf{u}$, is equivalent to *walk generating function* of the graph G, denoted by $N(G,\alpha)$ (cf. Cvetkovic, 1995). To see this, let $N_k(G) \equiv \sum_{i=1}^{n} N_{k,i}(G)$ denote the number of walks of length k in G. Then we can write $N_k(G)$ as follows $N_k(G) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{[k]} = \mathbf{u}^\top \mathbf{A}^k \mathbf{u}$. The walk generating function is then defined as

$$N(G,\alpha) \equiv \sum_{k=0}^{\infty} N_k(G)\alpha^k = \mathbf{u}^\top \left(\sum_{k=0}^{\infty} \alpha^k \mathbf{A}^k \right) \mathbf{u} = \mathbf{u}^\top \left(\mathbf{I}_n - \alpha\mathbf{A}\right)^{-1} \mathbf{u} = \mathbf{u}^\top \mathbf{M}(G,\alpha)\mathbf{u}.$$

Moreover, the generating function of the number of closed walks that start and terminate at node i is given by

$$W_i(G,\alpha) \equiv \sum_{k=0}^{\infty} a_{ii}^{[k]} \alpha^k. \tag{61}$$

The Bonacich matrix of equation (58) is also a measure of structural similarity of the agents in the network, called *regular equivalence*. Blondel *et al.* (2004) and Leicht *et al.* (2006) define a similarity score b_{ij}, which is high if nodes i and j have neighbors that themselves have high similarity, given by $b_{ij} = \alpha \sum_{k=1}^{n} a_{ik} b_{kj} + \delta_{ij}$. In matrix-vector notation this reads $\mathbf{M} = \alpha\mathbf{A}\mathbf{M} + \mathbf{I}$. Rearranging yields $\mathbf{M} = (\mathbf{I} - \alpha\mathbf{A})^{-1} = \sum_{k=0}^{\infty} \alpha^k \mathbf{A}^k$, assuming that $\alpha < 1/\lambda_{\max}$. We hence obtain

[50] The proof can be found e.g. in Debreu and Herstein (1953).

that the similarity matrix $\mathbf{M}$ is equivalent to the Bonacich matrix from equation (58). The average similarity of agent i is $\frac{1}{n}\sum_{j=1}^{n} b_{ij} = \frac{1}{n} b_{\mathbf{u},i}(G,\alpha)$, where $b_{\mathbf{u},i}(G,\alpha)$ is the Bonacich centrality of i. It follows that the Bonacich centrality of i is proportional to the average regular equivalence of i. Agents with a high Bonacich centrality are then the ones which also have a high average structural similarity with the other agents in the network.

The interpretation of eingenvector-like centrality measures as a similarity index is also important in the study of correlations between observations in principal component analysis and factor analysis (cf. Rencher and Christensen, 2012). Variables with similar factor loadings can be grouped together. This basic idea has also been used in the economics literature on segregation (e.g. Ballester and Vorsatz, 2014; Echenique and Fryer Jr., 2007; Echenique *et al.*, 2006).

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